MAT 145 - Problem Set 4
Due beginning of lecture on October 31st

Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Prove that if we move straight down in Pascal’s Triangle (visiting every other row), then the numbers we see are increasing.

2. Prove that
\[1 + \binom{n}{2} 2 + \binom{n}{4} 4 + \cdots + \binom{n}{n-1} 2^{n-1} + \binom{n}{n} 2^n = 3^n.\]

3. Prove that
\[\sum_{n_1+n_2+\cdots+n_k=n} \binom{n}{n_1, n_2, \ldots, n_k} = k^n\]

4. In how many ways can you cover a $2 \times n$ chessboard by dominoes? (Hint: Fibonacci recurrence)

5. Assume that the sequence $(a_0, a_1, a_2, \ldots)$ satisfies the recurrence $a_{n+1} = a_n + 2a_{n-1}$. We know that $a_0 = 4$ and $a_2 = 13$. What is $a_5$?

6. Prove that if $n$ is a multiple of 4, then $F_n$ (the $n$th Fibonacci number) is a multiple of 3.