Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Give an implementation-level description and a formal description (i.e. including the state diagram of the transition function) of a TM that recognizes
\[
\{ u\#v : u, v \in \{0, 1\}^* \text{ and } u \text{ is } v \text{ reversed} \}
\]

2. Show that the collection of decidable languages is closed under concatenation.
(The concatenation of two languages \(L, M\) is the language \(\{vw : v \in L, w \in M\}\))

3. * Show that a language is decidable iff there is an enumerator that prints it out in lexicographic order.

4. * Let \(C\) be a language. Prove that \(C\) is Turing-recognizable iff a decidable language \(D\) exists such that \(C = \{x : \exists y(\langle x, y \rangle \in D)\}\).

5. Let \(T = \{\langle M \rangle : M\) is a T.M. that accepts \(w\) reversed whenever it accepts \(w\}.\)
   Show that \(T\) is undecidable.

6. Show that \(A\) is Turing-recognizable iff \(A \leq_m A_{TM}\).