

# CSE 6321 - Problem Set 3

## Due lecture on April 18th

Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Show that if every NP-hard language is also PSPACE-hard, then  $PSPACE = NP$ .
2. Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.
3. Read the definition of MIN-FORMULA from Problem set 2.

(a) Show that  $MIN - FORMULA \in PSPACE$ .

(b) Explain why this argument fails to show that  $MIN - FORMULA \in coNP$ : If  $\phi \notin MIN - FORMULA$ , then  $\phi$  has a smaller equivalent formula. A NTM can verify that  $\phi \in \overline{MIN - FORMULA}$  by guessing that formula.

4. An undirected graph is *bipartite* if its nodes may be divided into two sets so that all edges go from a node in one set to a node in the other set. Show that a graph is bipartite if and only if it doesn't contain a cycle that has an odd number of nodes. Let  $BIPARTITE = \{ \langle G \rangle : G \text{ is a bipartite graph} \}$ . Show that  $BIPARTITE \in NL$ .
5. Recall that a directed graph is *strongly connected* if every two nodes are connected by a directed path in each direction. Let

$STRONGLY - CONNECTED = \{ \langle G \rangle : G \text{ is a strongly connected graph} \}$ .

Show that  $STRONGLY - CONNECTED$  is NL-complete.