Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Prove that the union of countably many countable sets is countable.

2. Let $C$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists such that $C = \{x : \exists y (\langle x, y \rangle \in D)\}$.

3. Let $T = \{\langle M \rangle : M$ is a T.M. that accepts $w$ reversed whenever it accepts $w\}$. Show that $T$ is undecidable. Is $T$ recognizable?

4. Show that $A$ is Turing-recognizable iff $A \leq_m A_{TM}$. 