CSE 6321 - Problem Set 3
Due beginning of lecture on March 5th

Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. A *permutation* on the set $\{1, \ldots, k\}$ is a one-to-one, onto function on this set. When p is a permutation, p^t means the composition of p with itself t times. Let

   $$\text{PERM-POWER} = \{\langle p, q, t\rangle : p = q^t \text{ where } p \text{ and } q \text{ are permutations on } \{1, \ldots, k\} \text{ and } t \text{ is a binary integer}\}.$$

   Show that PERM-POWER $\in P$. (Note that the most obvious algorithm doesn't run within polynomial time. Hint: First try it where t is a power of 2.)

2. Prove that the following language is undecidable:

   $$A = \{\langle M\rangle : M \text{ is a TM that runs in time } 2^{O(n)}\}.$$

3. Let $coNP$ be the class of languages whose complement is in NP. Show that $P \subseteq NP \cap coNP$. Show that if $P = NP$ then $P = coNP$. (Warning: $coNP$ is *not* the complement of NP.)

4. Let

   $$DOUBLE-SAT = \{\langle \phi\rangle : \phi \text{ is a boolean formula that has at least two satisfying assignments }\}.$$

   Show that $DOUBLE-SAT$ is NP-complete.