Problem numbers are from the third edition of Sipser’s book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited. Printed version is preferred, otherwise please make sure your handwriting is readable.

1. 3.7

2. Prove that the union of countably many countable sets is countable.

3. Let $C \subseteq \Sigma^*$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists such that $C = \{x \in \Sigma^* : (\exists y \in \Sigma^*) (x, y) \in D\}$.

4. Let $T = \{\langle M \rangle : M$ is a T.M. that accepts $w$ reversed whenever it accepts $w\}$. Determine whether $T$ is decidable, undecidable but recognizable or unrecognizable. Prove your answer.