

CSE 6321 - Problem Set 3

Due beginning of lecture on February 23rd

Problem numbers are from the third edition of Sipser's book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited. Printed version is preferred, otherwise please make sure your handwriting is readable.

1. Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.
2. Let

$$M = \{ \langle a, b, c, d \rangle : a, b, c \text{ and } p \text{ are binary integers} \\ \text{such that } a^b \equiv c \pmod{p} \}.$$

Show that $M \in P$. (Note that the most obvious algorithm does not run in polynomial time. Hint: Try first where b is a power of 2.)

3. Prove that the following language is undecidable:

$$A = \{ \langle M \rangle : M \text{ is a TM with running time } O(n) \}.$$

4. Prove that the following language is undecidable:

$$A = \{ \langle M \rangle : L(M) \in \text{TIME}(n) \}.$$