CSE 2331 Homework 1
Spring, 2016

1. Give the asymptotic complexity of each of the following functions in simplest terms. Your solution should have the form $\Theta(n^\alpha)$ or $\Theta((\log_\mu(n))^\beta)$ or $\Theta(n^\alpha(\log_\mu(n))^\beta)$ or $\Theta(\gamma^{\delta n})$ or $\Theta(1)$ where $\alpha, \beta, \gamma, \delta, \mu$ are constants. (No need to give any justification or proof.)

   (a) $f_a(n) = 3\log_4(n^3 + n^2) + 4n^{0.4}$;
   (b) $f_b(n) = \log_4(2n^2) \times \log_3(6n^{0.1} + 8) + \log_6(5n^3 + n^2 + 8)$;
   (c) $f_c(n) = 7n^{0.6} + 3\sqrt{n}$;
   (d) $f_d(n) = 2\sqrt{5n^3 + 8n^2 + 7n + 1}$;
   (e) $f_e(n) = 17\log_6(3n + 15) + 22\log_{20}(5n + 9)$;
   (f) $f_f(n) = 2^{12} + 3^3 \times 6\log_5(88888)$;
   (g) $f_g(n) = 8\log_4(5n^3 - 9n^2) + 7\log_3(8n^4 - 5n^3)$;
   (h) $f_h(n) = \sqrt{5\log_3(n^2) + 12n + 18}$;
   (i) $f_i(n) = 5n\log_6(4n^2 + 15n + 8) + 7n$;
   (j) $f_j(n) = 6\log_5(n) + \sqrt{8n} + 7n$;
   (k) $f_k(n) = 6\log_4(7^n + n^7 + 5)$;
   (l) $f_l(n) = 4(n + 11)\log_4(5n^3 + 16n^2) + 10n + 2^8$;
   (m) $f_m(n) = 7^n + 10^n + 12^n$;
   (n) $f_n(n) = 5^{2n} + 6 \times 5^n$;
   (o) $f_o(n) = 4n^5 + 3^{n+7} + 3^{n+2}$;
   (p) $f_p(n) = 6 \times 7^{n+8} + 100 \times 4^{n+8}$;
   (q) $f_q(n) = \sqrt{4n^2 + 8n + 16}$;
   (r) $f_r(n) = 9n^4 + 3^{2n} + 6 \times 7^n$;
   (s) $f_s(n) = 8 \times 5^{\log_5(n^3+8n^2)}$;
   (t) $f_t(n) = (2\log_5(n^5 + n^3) + 7\sqrt{n + 9}) \times (5\log_3(3n + 9) + 13\log_2(8n + 6))$;
   (u) $f_u(n) = 6\log_5(4 + \sqrt{3n + 8})$;

2. Give an example of a function $f(n)$ such that:
   - $f(n) \in O(n^2)$ and $f(n) \in \Omega(n\sqrt{n})$ but $f(n) \notin \Theta(n^2)$ and $f(n) \notin \Theta(n\sqrt{n})$.

3. Give an example of a function $f(n)$ such that:
   - $f(n) \in O(\sqrt{n})$ and $f(n) \in \Omega(\log_2(n))$ but $f(n) \notin \Theta(\sqrt{n})$ and $f(n) \notin \Theta(\log_2(n))$.

4. Prove that $4\sqrt{3n^6 - 8n^5 + 17} \in \Theta(n^3)$ using the definition of $\Theta(n^3)$ as functions $f(n)$ such that $c_1n^3 \leq f(n) \leq c_2n^3$ for constants $c_1, c_2 \geq 0$ for all large n.

5. Let $f(n) = 3\sqrt{6n + 5}(\log_8(5n + 9))^3$ and $g(n) = 6\sqrt{11n}\log_5(3n^2 + 8n) \times 15\log_3(7n + 19)$. Prove that $f(n) \in \Omega(g(n))$ using $\lim_{n\to\infty} f(n)/g(n)$.