For each of the following problems, simplify and express your answer as $\Theta(n^k)$ or $\Theta(n^k(\log n))$ wherever possible. If the asymptotic running time is exponential, then just give exponential lower bounds.

Random(n) generates a random number between 1 and n with uniform distribution (every integer between 1 and n is equally likely.) **CoinFlip**$()$ returns **heads** or **tails** with equal probability.

1. Consider the following function:

   ```
   func1(A, n)
   /* A is an array of integers                                    */
   1 s ← 0;
   2 k ← Random(n);
   3 for i ← 1 to k do
   4   |  for j ← 1 to ⌊k^0.2⌋ do
   5   |  |   s ← s + A[i] * A[j];
   6   |  end
   7 end
   8 return (s);
   ```

   (a) What is the asymptotic worst case running time of `func1`?

   (b) What is the asymptotic expected running time of `func1`? Justify your solution.

2. Consider the following function:

   ```
   func2(A, n)
   /* A is an array of integers                                     */
   1  s ← 0;
   2  for i ← 1 to n do
   3    |  k ← Random(i);
   4    |  for j ← 1 to k do
   5    |  |   for m ← 1 to k do
   6    |  |  |    s ← s + A[i] * A[j] * A[m];
   7    |  |  end
   8    |  end
   9  end
   10 return (s);
   ```

   (**Random**(i) generates a random number between 1 and i with uniform distribution (every integer between 1 and i is equally likely.)

   (a) What is the asymptotic worst case running time of `func2`?

   (b) What is the asymptotic expected running time of `func2`? Justify your solution.

3. Consider the following function:

```
func3(A, n)
/* A is an array of integers                                          */
1  s ← 0;
2  k ← Random(n);
3  if (k = 1) then
4      for i ← 1 to n do
5          for j ← 1 to ⌊√n⌋ do
6          |   s ← s + A[i] * A[2(j)];
7          end
8      end
9  else
10     for i ← 1 to ⌊n^0.75⌋ do
11     |   s ← s + A[i];
12     end
13 end
14 return (s);
```

(a) What is the asymptotic worst case running time of `func3`?

(b) What is the asymptotic expected running time of `func3`? Justify your solution.

4. Consider the following function:

```
func4(A, n)
/* A is an array of integers                                          */
1  s ← 0;
2  k ← Random(n);
3  if (k < n^0.4) then
4      for i ← 1 to ⌊n/2⌋ do
5          for j ← 1 to ⌊√n⌋ do
6          |   s ← s + A[i] * A[2(j)];
7          end
8      end
9  else
10     for i ← 1 to ⌊n^0.75⌋ do
11     |   s ← s + A[i];
12     end
13 end
14 return (s);
```

(a) What is the asymptotic worst case running time of `func4`?

(b) What is the asymptotic expected running time of `func4`? Justify your solution.

5. Consider the following function:

```
func5(A, n)
/* A is an array of integers                                               */
1 if (n ≤ 100) then return (A[1]);
2 s ← A[1];
3 for i ← 1 to n do
4     for j ← 1 to ⌊log₂ n⌋ do
5         │ s ← s + A[i] * A[i * j];
6     end
7 end
8 k₁ ← Random (n);
9 k₂ ← Random (n);
10 if (k₁ < k₂) then  s ← s + func5(A, n − 5) ;
11 return (s);
```

(a) What is the asymptotic worst case running time of func5? Justify your solution.

(b) What is the asymptotic expected running time of func5? Justify your solution.

6. Consider the following function:

```
func6(A, n)
/* A is an array of integers                                               */
1 if (n ≤ 10) then return (A[1]);
2 i ← 2;
3 while (i < n⁴) do
4     k ← Random(n);
5     if (k < (2/3)n) then  return (A[i]) ;
6     i ← i + 4;
7 end
8 return (A[n]);
```

(a) What is the asymptotic worst case running time of func6? Justify your solution.

(b) What is the asymptotic expected running time of func6? Justify your solution.