Problem numbers are from the third edition of Sipser’s book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited.

1. Give an implementation-level description and a formal description (i.e. including the state diagram of the transition function) of a TM that recognizes

\[ \{ u \# v : u, v \in \{0,1\}^* \text{ and } u \text{ is } v \text{ reversed} \} \]

2. Show that the collection of decidable languages is closed under the operations of

   (a) complementation.
   (b) intersection.

3. Show that the collection of recognizable languages is closed under the operations of

   (a) concatenation (Given languages \( A, B \), their concatenation is \( A \circ B := \{ xy : x \in A, y \in B \} \), where \( xy \) is the concatenation of strings \( x, y \)).
   (b) intersection.

4. Show that a language is decidable iff there is an enumerator that prints it out in standard string order. (Standard string order: shorter strings come before longer strings, strings of the same length come in alphabetic order. In the 3rd edition of the book this is also called shortlex order. In earlier editions this is called lexicographic order.)