CSE 6321 - Problem Set 2
Due beginning of lecture on February 16th

Problem numbers are from the third edition of Sipser's book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited. Printed version is preferred, otherwise please make sure your handwriting is readable.

1. Let $C \subseteq \Sigma^*$ be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that $C = \{x \in \Sigma^* : (\exists y \in \Sigma^*)\langle x, y\rangle \in D\}$.

2. Let $T = \{\langle M\rangle : M$ is a T.M. that accepts w reversed whenever it accepts $w.\}$. Determine whether T is decidable, undecidable but recognizable or unrecognizable. Prove your answer.

3. Let

$$M = \{\langle a, b, c, d\rangle : a, b, c \text{ and } p \text{ are binary integers}$$
$$\text{such that } a^b \equiv c \mod p\}.$$

   Show that $M \in P$. (Note that the most obvious algorithm does not run in polynomial time. Hint: Try first where b is a power of 2.)

4. Prove that the following language is undecidable:

$$A = \{\langle M\rangle : M \text{ is a TM with running time } O(n)\}.$$

5. Prove that the following language is undecidable:

$$A = \{\langle M\rangle : L(M) \in \text{TIME}(n)\}.$$