CSE 6321 - Problem Set 5
Due beginning of lecture on March 29th

Problem numbers are from the third edition of Sipser's book. If unsure about which problem to solve, ask. Collaboration is permitted; looking for solutions from external sources (books, the web, material from previous years, etc.) is prohibited. Printed version is preferred, otherwise please make sure your handwriting is readable.

1. Consider the optimization problem (Q) of the form

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^{\mathrm{T}}P_0x + q_0^{\mathrm{T}}x \\ \text{subject to} & \frac{1}{2}x^{\mathrm{T}}P_ix + q_i^{\mathrm{T}}x + r_i \leq 0 \quad \text{for } i = 1, \ldots, m, \\ & Ax = b, x \in \mathbb{R}^n \end{array}$$

   where the inputs are the binary encodings of $A \in \mathbb{Q}^{k\times n}$, $P_0, P_1, \ldots, P_m \in \mathbb{Q}^{n\times n}$, $b \in \mathbb{Q}^k$, $q_0, q_1, \ldots, q_m \in \mathbb{Q}^n, r_1, \ldots, r_m \in \mathbb{Q}$. Show an equivalent decision version of (Q).

2. Show that (Q) from problem 1 is NP-hard.

   Hint: Let D-Q be an equivalent decision version. Show that for some NP-complete problem L we have $L \leq_P$ D-Q.

3. Show that if every NP-hard language is also PSPACE-hard, then PSPACE = NP. (Clarification: A language A is PSPACE-hard if for every language $B \in$ PSPACE we have $B \leq_P A$. A language A is NP-hard if for every language $B \in$ NP we have $B \leq_P A$).

4. Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.