MATH 108

DISCUSSION SHEET 1

1. WRITE THE FOLLOWING AS "IF, THEN" STATEMENTS.
   A) THE SQUARE OF EVERY ODD INTEGER IS ODD.
   B) THE INTEGER n^3 IS EVEN ONLY IF n IS EVEN.
   C) A SUFFICIENT CONDITION FOR A TRIANGLE TO BE ISOSCELES IS THAT IT HAS TWO EQUAL ANGLES.

2. USE A TRUTH TABLE TO SHOW THAT $\sim P \Rightarrow (Q \wedge \sim Q)$ IS EQUIVALENT TO P.

3. PROVE THAT IF a AND c ARE ODD INTEGERS,
   THEN $ab + bc$ IS EVEN FOR EVERY INTEGER b.

4. WRITE THE FOLLOWING VERSION OF THE ARCHIMEDEAN PRINCIPLE IN SYMBOLS:
   "FOR EVERY REAL NUMBER X, THERE IS A NATURAL NUMBER n SUCH THAT $n > X$."

5. WRITE THE FOLLOWING STATEMENTS IN SYMBOLS, AND THEN FIND A NEGATION FOR EACH OF THEM (WHICH HAS NO NEGATION SYMBOLS):
   A) FOR EVERY $X > 0$ THERE IS A $Y > 0$ SUCH THAT $Y < \frac{1}{X}$.
   B) FOR EVERY $\varepsilon > 0$ THERE IS A $\delta > 0$ SUCH THAT IF $|X - c| < \delta$, THEN $|f(x) - f(c)| < \varepsilon$.
   C) FOR EVERY $\varepsilon > 0$ THERE IS A POSITIVE INTEGER N SUCH THAT IF $n \geq N$, THEN $|f_n(x) - f(x)| < \varepsilon$ FOR ALL X IN J.

6. DEFINE THE "<u>SHEFFER STROKE</u>" BY $\underline{P \uparrow Q}$ REPRESENTS $\underline{\sim (P \wedge Q)}$.
   ALL THE LOGICAL CONNECTIVES $\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$ CAN BE DEFINED IN TERMS OF THIS SINGLE CONNECTIVE:
   A) REPRESENT $\underline{\sim P}$ USING THIS SYMBOL.
   B) REPRESENT $\underline{P \wedge Q}$ USING THIS SYMBOL.
   C) REPRESENT $\underline{P \vee Q}$ USING THIS SYMBOL.