MATH 108 GOLD SHEET

① LET $A = \{1, 2, 3, 4\}$.

a) FIND THE NUMBER OF RELATIONS ON A.

b) FIND THE NUMBER OF FUNCTIONS $f: A \to A$.

c) FIND THE NUMBER OF BIJECTIONS $f: A \to A$.

d) FIND THE NUMBER OF EQUIVALENCE RELATIONS ON A.
[HINT: USE THE IDEAS IN SEC. 3.3.]

② USE THE FOLLOWING STEPS TO PROVE EUCLID'S LEMMA:

> LET p BE A PRIME AND LET $a, b \in \mathbb{N}$.
> IF $p \mid ab$, THEN $p \mid a$ OR $p \mid b$.

A) ASSUME THAT $p \mid ab$ AND $p \nmid a$. WE MUST SHOW THAT $p \mid b$.

B) LET $T = \{n \in \mathbb{N} : p \mid nb\}$.
SHOW THAT $a \in T$ AND $p \in T$, AND CONCLUDE BY THE WOP THAT T MUST HAVE A LEAST ELEMENT c.

C) USE THE DIVISION ALGORITHM TO SHOW THAT $c \mid n$ FOR EVERY $n \in T$.

D) USE THIS FACT TO SHOW THAT $c = 1$, AND CONCLUDE THAT $p \mid b$.

③ DEFINE $f: (-1, 1) \to \mathbb{R}$ BY $f(x) = \dfrac{x}{1 - x^2}$.

SHOW THAT f IS A BIJECTION.