MATH 108 HOMEWORK SOLUTIONS (Sec. 1.1 - 1.2)

1.1 - (5) f)

P	Q	R	Q∨R	P∧Q	P∧R	P∧(Q∨R)	(P∧Q)∨(P∧R)
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

i)

P	Q	P∨Q	∼(P∨Q)	∼P	∼Q	∼P∧∼Q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

(11) f) K IS NOT BOUNDED OR K IS COMPACT. $(\sim P \vee Q)$

DENIAL: K IS BOUNDED AND K IS NOT COMPACT. $(P \wedge \sim Q)$

j) NEITHER $Z < 5$ NOR $Z \leq T$ IS TRUE. $(\sim P \wedge \sim Q)$

DENIAL: EITHER $Z \geq 5$ OR $Z > T$. $(P \vee Q)$

1.2 - (5) b) IF HEXAGONS HAVE SIX SIDES, THEN THE MOON IS MADE OF CHEESE.

FALSE $(P \Rightarrow Q$ WITH P TRUE AND Q FALSE$)$

c) IF EUCLID'S BIRTHDAY WAS APRIL 2, THEN RECTANGLES HAVE 4 SIDES.

TRUE $(P \Rightarrow Q$ WITH Q TRUE$)$

(6) g) $x^2 \geq 0$ IFF $x \geq 0$ (F IF $x < 0$, AND T IF $x \geq 0$)

h) $x^2 - y^2 = 0$ IFF $(x-y)(x+y) = 0$ (T FOR ANY VALUES OF X AND Y)

(12) b) $(P \wedge Q) \Rightarrow R \equiv (P \wedge \sim R) \Rightarrow \sim Q$

PF $(P \wedge Q) \Rightarrow R \equiv \sim(P \wedge Q) \vee R \equiv (\sim P \vee \sim Q) \vee R \equiv ((\sim P) \vee R) \vee (\sim Q)$

$\equiv \sim(P \wedge \sim R) \vee (\sim Q) \equiv (P \wedge \sim R) \Rightarrow \sim Q$

(USING TH. 1.1.1 AND TH. 1.2.2)

d) $P \Rightarrow (Q \vee R) \equiv (P \wedge \sim R) \Rightarrow Q$

PF $P \Rightarrow (Q \vee R) \equiv \sim P \vee (Q \vee R) \equiv (\sim P \vee R) \vee Q$

$\equiv \sim(P \wedge \sim R) \vee Q \equiv (P \wedge \sim R) \Rightarrow Q$

(USING TH. 1.1.1 AND TH. 1.2.2)

MATH 108 HOMEWORK SOLUTIONS (Sec. 1.2)

(15b) IF n IS PRIME, THEN $n=2$ OR n IS ODD.

CONVERSE: IF $n=2$ OR n IS ODD, THEN n IS PRIME. (FALSE: TAKE $n=9$.)

CONTRAPOSITIVE: IF $n\neq 2$ AND n IS EVEN, THEN n IS NOT PRIME. (TRUE)

REMARK HERE THERE IS AN IMPLICIT UNIVERSAL QUANTIFIER: $(\forall n\in\mathbb{N})$.

(16a) $((P\Rightarrow Q)\Rightarrow P)\Rightarrow P$ IS A TAUTOLOGY:

P	Q	$P\Rightarrow Q$	$(P\Rightarrow Q)\Rightarrow P$	$((P\Rightarrow Q)\Rightarrow P)\Rightarrow P$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

(16c) $P\Rightarrow Q \Leftrightarrow P\wedge(\sim Q)$ IS A CONTRADICTION:

$P\Rightarrow Q \equiv \sim P\vee Q$, AND $\sim(\sim P\vee Q)\equiv P\wedge(\sim Q)$;

SO THIS IS OF THE FORM $S\Leftrightarrow \sim S$ WHERE S IS $P\Rightarrow Q$.

OR

P	Q	$P\Rightarrow Q$	$\sim Q$	$P\wedge(\sim Q)$	$(P\Rightarrow Q)\Leftrightarrow(P\wedge(\sim Q))$
T	T	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F