MATH 108 SEC. 2.2 - SOLUTIONS

① d) $A \cup B - C \cap D = \{0, 3, 4, 6, 9\}$

② e) $B - D = [2, 5]$

⑦ n) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

PF $x \in A \cup (B \cap C)$ IFF $x \in A$ OR $x \in B \cap C$ IFF $x \in A$ OR ($x \in B$ AND $x \in C$)
IFF ($x \in A$ OR $x \in B$) AND ($x \in A$ OR $x \in C$) IFF $x \in A \cup B$ AND $x \in A \cup C$
IFF $x \in (A \cup B) \cap (A \cup C)$, SO $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

o) $A \subseteq B$ IFF $A \cup B = B$

PF $\Leftarrow$ SUPPOSE $A \cup B = B$. SINCE $A \subseteq A \cup B$, $A \subseteq B$.

$\Rightarrow$ SUPPOSE $A \subseteq B$. THEN 1) $B \subseteq A \cup B$, AND
2) IF $x \in A \cup B$, THEN $x \in A$ OR $x \in B$.
IF $x \in A$, THEN $x \in B$ SINCE $A \subseteq B$; SO $x \in B$ IN EITHER CASE.
THEREFORE $A \cup B \subseteq B$; SO $A \cup B = B$.

⑧ f) $A \cap B = \emptyset$ IFF $A \subseteq B^c$.

PF $\Rightarrow$ ASSUME THAT $A \cap B = \emptyset$. IF $x \in A$, THEN $x \notin B$ SO $x \in B^c$;
AND THEREFORE $A \subseteq B^c$.

$\Leftarrow$ ASSUME THAT $A \subseteq B^c$. IF $x \in A \cap B$, THEN $x \in A$ SO $x \in B^c$ SINCE
$A \subseteq B^c$; AND THIS GIVES A CONTRADICTION SINCE $x \in B$.
THEREFORE $A \cap B = \emptyset$.

h) $(A \cap B)^c = A^c \cup B^c$

PF $x \in (A \cap B)^c$ IFF $x \notin A \cap B$ IFF IT IS NOT TRUE THAT $x \in A$ AND $x \in B$
IFF $x \notin A$ OR $x \notin B$ IFF $x \in A^c$ OR $x \in B^c$ IFF $x \in A^c \cup B^c$.
THEREFORE $(A \cap B)^c = A^c \cup B^c$.

⑨ b) IF $A \subseteq B \cup C$ AND $A \cap B = \emptyset$, THEN $A \subseteq C$.

PF LET $x \in A$. SINCE $A \subseteq B \cup C$, $x \in B \cup C$ SO $x \in B$ OR $x \in C$.
IF $x \in B$, THEN $x \in A \cap B = \emptyset$, WHICH GIVES A CONTRADICTION;
SO $x \in C$. THEREFORE $A \subseteq C$.

MATH 108

Sec. 2.2 – Solutions

(9) e) $(A-B)-C = (A-C)-(B-C)$

PF 1) Let $x \in (A-B)-C$, so $x \in A-B$ and $x \notin C$ and therefore $x \in A$ and $x \notin B$ and $x \notin C$.

Since $x \in A$ and $x \notin C$, $x \in A-C$; and since $x \notin B$, $x \notin B-C$.

Therefore $x \in (A-C)-(B-C)$; so $(A-B)-C \subseteq (A-C)-(B-C)$.

2) Let $x \in (A-C)-(B-C)$, so $x \in A-C$ and $x \notin B-C$.

Then $x \in A$ and $x \notin C$, and $x \notin B$ since otherwise $x \in B-C$.

Since $x \in A$ and $x \notin B$, $x \in A-B$; so $x \in (A-B)-C$ since $x \notin C$.

Therefore $(A-C)-(B-C) \subseteq (A-B)-C$.

By 1) and 2), $(A-B)-C = (A-C)-(B-C)$.

(10) d) If $C \subseteq A$ and $D \subseteq B$, then $D-A \subseteq B-C$.

PF Let $x \in D-A$, so $x \in D$ and $x \notin A$.

Since $x \in D$ and $D \subseteq B$, $x \in B$.

Since $x \in A^c$ and $A^c \subseteq C^c$, $x \in C^c$ so $x \notin C$.

[or use that $x \in C$ and $C \subseteq A$ would imply $x \in A$, which gives a contradiction; so $x \notin C$.]

Therefore $x \in B-C$; so $D-A \subseteq B-C$.

e) If $A \cup B \subseteq C \cup D$, $A \cap B = \emptyset$, and $C \subseteq A$, then $B \subseteq D$.

PF Let $x \in B$, so $x \in A \cup B$ and therefore $x \in C \cup D$ since $A \cup B \subseteq C \cup D$.

Since $x \in C \cup D$, either $x \in C$ or $x \in D$.

If $x \in C$, then $x \in A$ since $C \subseteq A$; and this gives a contradiction since $x \in A \cap B$ with $A \cap B = \emptyset$.

Therefore $x \in D$; so $B \subseteq D$.

(16) a) Let $A = \{x\}$, $B = \{0\}$, $C = \{y\}$, $D = \{1\}$.

Then $(A \times B) \cup (C \times D) \neq (A \cup C) \times (B \cup D)$ since

$(A \times B) \cup (C \times D) = \{(x,0), (y,1)\}$ and $(A \cup C) \times (B \cup D) = \{(x,0), (x,1), (y,0), (y,1)\}$.

(20) e) F – The claim is false: for example, let $A = \{1,2\}$, $B = \{2,3\}$, $C = \{3,4\}$.

(The element $x \in A \cap B$ is not the same as the element $x \in B \cap C$, so different symbols must be used for these elements.)