MATH 108 — SEC. 2.3 – SOLUTIONS

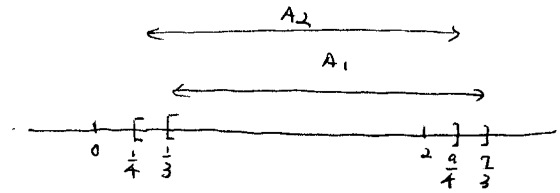
(1) d) $B_n = \mathbb{N} - \{1, \dots, n\}$ FOR $n \in \mathbb{N}$

$B_1 = \{2,3,4,5,\dots\}$
$B_2 = \{3,4,5,\dots\}$
$B_3 = \{4,5,6,\dots\}$

$\bigcup_{n\in\mathbb{N}} B_n = \boxed{\mathbb{N} - \{1\}}$ $\qquad \bigcap_{n\in\mathbb{N}} B_n = \boxed{\emptyset}$

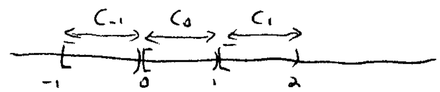
k) $A_n = \left[\frac{1}{n}, 2 + \frac{1}{n}\right]$ FOR $n \geq 3$

$\bigcup_{n\in\mathbb{N}} A_n = \boxed{\left(0, \frac{7}{3}\right]}$ $\qquad \bigcap_{n\in\mathbb{N}} A_n = \boxed{\left[\frac{1}{3}, 2\right]}$



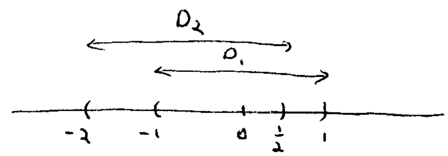
l) $C_n = [n, n+1)$ FOR $n \in \mathbb{Z}$

$\bigcup_{n\in\mathbb{Z}} C_n = \boxed{\mathbb{R}}$ $\qquad \bigcap_{n\in\mathbb{N}} C_n = \boxed{\emptyset}$



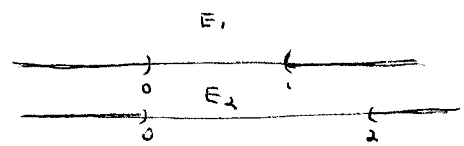
n) $D_n = \left(-n, \frac{1}{n}\right)$ FOR $n \in \mathbb{N}$

$\bigcup_{n\in\mathbb{N}} D_n = \boxed{(-\infty, 1)}$ $\qquad \bigcap_{n\in\mathbb{N}} D_n = \boxed{(-1, 0]}$



r) $E_n = \mathbb{R} - [0, n]$ FOR $n \in \mathbb{N}$

$\bigcup_{n\in\mathbb{N}} E_n = \boxed{\mathbb{R} - [0, 1]}$ $\qquad \bigcap_{n\in\mathbb{N}} E_n = \boxed{(-\infty, 0)}$



(4) a) $A_n = \{4n, 4n+1, \dots, 5n\}$ FOR EACH $n \in \mathbb{N}$.

(1) $\bigcup_{n\in\mathbb{N}} A_n = \{n \in \mathbb{N} : n \geq 4\} - \{6, 7, 11\}$

PF LET $B = \{n \in \mathbb{N} : n \geq 4\} - \{6, 7, 11\}$.

1) IF $m \in \bigcup_{n\in\mathbb{N}} A_n$, THEN $m \in A_n$ FOR SOME $n \in \mathbb{N}$ AND THEREFORE $4n \leq m \leq 5n$.
SINCE $n \geq 1$, $m \geq 4$; AND $m \notin \{6, 7, 11\}$ SINCE

$m = 6 \implies 4n \leq 6 \leq 5n$ FOR SOME $n \in \mathbb{N} \implies n = 1 \implies 6 \leq 5$, WHICH GIVES A CONTRADICTION;

$m = 7 \implies 4n \leq 7 \leq 5n$ FOR SOME $n \in \mathbb{N} \implies n = 1 \implies 7 \leq 5$, WHICH GIVES A CONTRADICTION; AND

$m = 11 \implies 4n \leq 11 \leq 5n$ FOR SOME $n \in \mathbb{N} \implies n \leq 2$ AND $n \geq 3$, WHICH GIVES A CONTRADICTION.

THEREFORE $\bigcup_{n\in\mathbb{N}} A_n \subseteq B$.

2) IF $m \in B$, THEN $m = 4n + r$ WHERE $n \geq 1$ SINCE $m \geq 4$ AND $0 \leq r < 3$,
USING THE DIVISION ALGORITHM.

i) IF $m \geq 12$, THEN $m \leq 4n + 3 \leq 5n$ SINCE $n \geq 3$; SO $m \in A_n$.

ii) IF $m \in \{4, 5\}$, THEN $m \in A_1$.

iii) IF $m \in \{8, 9, 10\}$, THEN $m \in A_2$

THEREFORE $B \subseteq \bigcup_{n\in\mathbb{N}} A_n$.

(2) $\bigcap_{n\in\mathbb{N}} A_n = \emptyset$ SINCE $x \in \bigcap_{n\in\mathbb{N}} A_n \implies x \in A_1 \cap A_2 = \emptyset$, WHICH GIVES A CONTRADICTION.

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4 c) $C_n = \{1-n, n-4\}$ FOR ALL $n \in \mathbb{N}$

1 $\bigcup_{n\in\mathbb{N}} C_n = \mathbb{Z}$

PF 1) SINCE $C_n \subseteq \mathbb{Z}$ FOR EACH n, $\bigcup_{n\in\mathbb{N}} C_n \subseteq \mathbb{Z}$.

2) LET $m \in \mathbb{Z}$.

a) IF $m > 0$, THEN $n = m+4 \in \mathbb{N}$ AND $m \in C_n$.

b) IF $m \leq 0$, THEN $n = 1-m \in \mathbb{N}$ AND $m \in C_n$.

THEREFORE $m \in C_n$ FOR SOME $n \in \mathbb{N}$ IN EITHER CASE, SO $\mathbb{Z} \subseteq \bigcup_{n\in\mathbb{N}} C_n$.

2 $\bigcap_{n\in\mathbb{N}} C_n = \emptyset$ SINCE $m \in \bigcap_{n\in\mathbb{N}} C_n \Rightarrow m \in C_1 \cap C_2 = \emptyset$, WHICH GIVES A CONTRADICTION.

7 b) $B \cup \left(\bigcap_{\alpha\in\Delta} A_\alpha\right) = \bigcap_{\alpha\in\Delta} (B \cup A_\alpha)$

PF $x \in B \cup \left(\bigcap_{\alpha\in\Delta} A_\alpha\right)$ IFF $(x \in B)$ OR $(x \in A_\alpha$ FOR ALL $\alpha \in \Delta)$

IFF FOR ALL $\alpha\in\Delta$, $(x \in B$ OR $x \in A_\alpha)$

IFF FOR ALL $\alpha\in\Delta$, $(x \in B \cup A_\alpha)$

IFF $x \in \bigcap_{\alpha\in\Delta} (B \cup A_\alpha)$, SO $B \cup \left(\bigcap_{\alpha\in\Delta} A_\alpha\right) = \bigcap_{\alpha\in\Delta} (B \cup A_\alpha)$.

9 a) F: LET $B = \{x, y, z\}$, $A_1 = \{x, y\}$, $A_2 = \{z\}$, $\Delta = \{1, 2\}$.

THEN $B - \bigcap_{i\in\Delta} A_i = B - \emptyset = B$ AND $\bigcap_{i\in\Delta} (B - A_i) = \emptyset$.

b) F: LET $B = \{x, y, z\}$, $A_1 = \{x, y\}$, $A_2 = \{z\}$, $\Delta = \{1, 2\}$.

THEN $B - \bigcup_{i\in\Delta} A_i = \emptyset$ AND $\bigcup_{i\in\Delta} (B - A_i) = B$.

c) T: $\left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B = \bigcap_{\alpha\in\Delta} (A_\alpha - B)$

PF $x \in \left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B$ IFF $x \in \bigcap_{\alpha\in\Delta} A_\alpha$ AND $x \notin B$

IFF (FOR ALL $\alpha\in\Delta$, $x \in A_\alpha$) AND $x \notin B$

IFF FOR ALL $\alpha\in\Delta$, $x \in A_\alpha - B$

IFF $x \in \bigcap_{\alpha\in\Delta} (A_\alpha - B)$, SO $\left(\bigcap_{\alpha\in\Delta} A_\alpha\right) - B = \bigcap_{\alpha\in\Delta} (A_\alpha - B)$

d) T: $\left(\bigcup_{\alpha\in\Delta} A_\alpha\right) - B = \bigcup_{\alpha\in\Delta} (A_\alpha - B)$

PF $x \in \left(\bigcup_{\alpha\in\Delta} A_\alpha\right) - B$ IFF $x \in \bigcup_{\alpha\in\Delta} A_\alpha$ AND $x \notin B$ IFF FOR SOME $\alpha\in\Delta$, $x \in A_\alpha$ AND $x \notin B$

IFF FOR SOME $\alpha\in\Delta$, $x \in A_\alpha - B$ IFF $x \in \bigcup_{\alpha\in\Delta} (A_\alpha - B)$, SO $\left(\bigcup_{\alpha\in\Delta} A_\alpha\right) - B = \bigcup_{\alpha\in\Delta} (A_\alpha - B)$

17 b) IF $A_j \subseteq A_i$ WHEN $j \geq i$, THEN $\bigcup_{i\in\mathbb{N}} A_i = A_1$

PF 1) IF $x \in A_1$, THEN $x \in \bigcup_{i=1}^{\infty} A_i$, SO $A_1 \subseteq \bigcup_{i=1}^{\infty} A_i$.

2) IF $x \in \bigcup_{i=1}^{\infty} A_i$, THEN $x \in A_i$ FOR SOME $i \geq 1$. THEREFORE $A_i \subseteq A_1$, SO $x \in A_1$.

THUS $\bigcup_{i=1}^{\infty} A_i \subseteq A_1$.

THEREFORE $\bigcup_{i=1}^{\infty} A_i = A_1$.