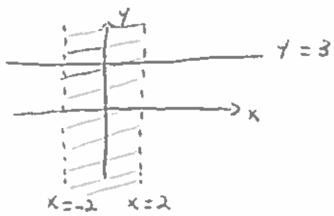
MATH 108 SOLUTIONS – 2.5, 3.1, AND 3.2

2.5 – (14) a) A

e) F – THIS CLAIM IS CLEARLY FALSE. THE INDUCTION STEP IS INVALID FOR $m=2$, SINCE $5^{m-2} \neq 5$.

3.1 – (2) a) $\text{Dom}(T) = \{1,2,3\}$ b) $\text{Rng}(T) = \{1,2,3,5,6\}$

(3) g) $x\,W\,y$ IFF $|x| < 2$ OR $y = 3$



$\text{Dom}(W) = \mathbb{R}$ (SINCE $x\,W\,3$ FOR ANY $x \in \mathbb{R}$)

$\text{Rng}(W) = \mathbb{R}$ (SINCE $0\,W\,y$ FOR ANY $y \in \mathbb{R}$)

(6) b) $R_2 = \{(x,y) \in \mathbb{R}\times\mathbb{R} : y = -5x+2\}$

$R_2^{-1} = \{(y,x) \in \mathbb{R}\times\mathbb{R} : y = -5x+2\} = \{(x,y) \in \mathbb{R}\times\mathbb{R} : x = -5y+2\}$

$= \{(x,y) \in \mathbb{R}\times\mathbb{R} : y = \frac{2-x}{5}\}$

3.2 – (1) d) $<$ ON $\mathbb{N}$: NOT REFLEXIVE, NOT SYMMETRIC, TRANSITIVE

f) $\neq$ ON $\mathbb{N}$: NOT REFLEXIVE, SYMMETRIC, NOT TRANSITIVE

k) $x\,S\,y$ IFF $3 \mid (x+y)$, ON $\mathbb{N}$:

1) NOT REFLEXIVE: $1 \not{S}\, 1$ SINCE $3 \nmid (1+1)$

2) SYMMETRIC: $x\,S\,y$ IFF $3 \mid (x+y)$ IFF $3 \mid (y+x)$ IFF $y\,S\,x$.

3) NOT TRANSITIVE: $1\,S\,2$ AND $2\,S\,4$, BUT $1 \not{S}\, 4$.

n) $(x,y)\,T\,(z,w)$ IFF $x+y \leq z+w$, ON $\mathbb{R}\times\mathbb{R}$:

1) REFLEXIVE: IF $(x,y) \in \mathbb{R}^2$, $(x,y)\,T\,(x,y)$ SINCE $x+y \leq x+y$.

2) NOT SYMMETRIC: $(0,0)\,T\,(1,1)$, BUT $(1,1) \not{T}\, (0,0)$.

3) TRANSITIVE: IF $(x,y)\,T\,(z,w)$ AND $(z,w)\,T\,(u,v)$, THEN $x+y \leq z+w$ AND $z+w \leq u+v \Rightarrow x+y \leq u+v \Rightarrow (x,y)\,T\,(u,v)$.

(2) b) REFLEXIVE, NOT SYMMETRIC, NOT TRANSITIVE:

$R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ (FOR EXAMPLE)

c) NOT REFLEXIVE, SYMMETRIC, NOT TRANSITIVE:

$R = \{(1,2), (2,3), (2,1), (3,2)\}$ (FOR EXAMPLE)

e) NOT REFLEXIVE, NOT SYMMETRIC, TRANSITIVE:

$R = \{(1,2), (2,3), (1,3)\}$ (FOR EXAMPLE)

MATH 108 SOLUTIONS — Sec. 3.2 AND 3.3

3.2 — *(3) d) REFLEXIVE, SYMMETRIC, NOT TRANSITIVE ON $\mathbb{R}$:

DEFINE $x R y$ IFF $xy \geq 0$, FOR EXAMPLE.

(THIS IS NOT TRANSITIVE SINCE $1 R 0$ AND $0 R -1$ BUT $1 \not R -1$, FOR INSTANCE).

$\boxed{\text{OR}}$ DEFINE $x R y$ IFF $xy < 0$ OR $x = y$, OR $x R y$ IFF $x = y$ OR $x = 2y$ OR $y = 2x$

f) REFLEXIVE, NOT SYMMETRIC, AND TRANSITIVE ON $\mathbb{R}$

DEFINE $x R y$ IFF $x \leq y$, FOR EXAMPLE.

g) NOT REFLEXIVE, SYMMETRIC, AND TRANSITIVE ON $\mathbb{R}$

DEFINE $x R y$ IFF $x \geq 0$ AND $y \geq 0$, FOR EXAMPLE.

$\boxed{\text{OR}}$ DEFINE $x R y$ IFF $xy > 0$. (THIS IS NOT REFLEXIVE SINCE $0 \not R 0$.)

(17) LET R BE A SYMMETRIC, TRANSITIVE RELATION ON A WITH $\text{Dom}(R) = A$.

THEN R IS REFLEXIVE.

PF IF $a \in A$, THEN $(a,b) \in R$ FOR SOME $b \in A$ SINCE $\text{Dom}(R) = A$.

SINCE R IS SYMMETRIC, $(b,a) \in R$; AND $(a,b) \in R$ AND $(b,a) \in R$ IMPLIES $(a,a) \in R$ SINCE R IS TRANSITIVE. THEREFORE R IS REFLEXIVE.

3.3 — (2) a) NOT A PARTITION, SINCE $\{1,2\} \neq \{2,3\}$ AND $\{1,2\} \cap \{2,3\} \neq \emptyset$.

b) NOT A PARTITION, SINCE $\bigcup_{X \in P} X = \{1,2,3,4,5\} \neq A$

c) THIS IS A PARTITION OF A.

(7) a) $\mathbb{N}$, $\{\{1,\ldots,9\}, \{10,\ldots,99\}, \{100,\ldots,999\}, \ldots\}$

THIS CORRESPONDS TO THE EQUIVALENCE RELATION

$x R y$ IFF x AND y HAVE THE SAME NUMBER OF DIGITS.

(11) $\mathcal{A} = \{R(x) : x \in A\}$ DOES NOT ALWAYS FORM A PARTITION OF A.

FOR EXAMPLE, LET $A = \{1,2,3\}$ AND

$R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (2,1), (3,2)\}$,

THEN $R(1) = \{1,2\}$, $R(2) = \{1,2,3\}$, AND $R(3) = \{2,3\}$;

AND $\mathcal{A} = \{\{1,2\}, \{1,2,3\}, \{2,3\}\}$ IS NOT A PARTITION OF A.

(SEE THE BACK OF THE BOOK FOR A SIMILAR EXAMPLE.)

* REMARK A SUBSET OF $\mathbb{R}^2$ GIVES A REFLEXIVE RELATION ON $\mathbb{R}$ IF IT CONTAINS THE LINE $Y = X$, AND IT GIVES A SYMMETRIC RELATION ON $\mathbb{R}$ IF IT IS SYMMETRIC AROUND THE LINE $Y = X$.