MATH 108 ANSWERS - QUIZ 1A

① PF (OF THE CONTRAPOSITIVE)

IF n IS ODD, THEN $n = 2k+1$ FOR SOME $k \in \mathbb{Z}$.

THEN $(n+4)^2 = (2k+5)^2 = 4k^2 + 20k + 25 = 2(2k^2 + 10k + 12) + 1$

WHERE $2k^2 + 10k + 12 \in \mathbb{Z}$, SO $(n+4)^2$ IS ODD.

② A) $p \Rightarrow q \equiv \underline{\sim p} \vee q$ B) $\sim(p \wedge q) \equiv (\sim p) \underline{\vee} (\sim q)$ C) $\sim(p \wedge q) \equiv p \underline{\Rightarrow} (\sim q)$

D) $p \Rightarrow q \vee r \equiv (p \wedge \underline{\sim q}) \Rightarrow r$ E) $\sim(p \Rightarrow q) \equiv p \underline{\wedge} (\sim q)$ F) $(p \wedge q) \vee (p \wedge r) \equiv \underline{p \wedge} (q \underline{\vee} r)$

③ PF 1) IF n IS EVEN, THEN $n = 2k$ FOR SOME $k \in \mathbb{Z}$; SO

$\underline{n^2 - 5n - 11} = 4k^2 - 10k - 11 = \underline{2(2k^2 - 5k - 6) + 1}$ IS ODD SINCE $2k^2 - 5k - 6 \in \mathbb{Z}$.

2) IF n IS ODD, THEN $n = 2k+1$ FOR SOME $k \in \mathbb{Z}$; SO

$\underline{n^2 - 5n - 11} = (2k+1)^2 - 5(2k+1) - 11 = 4k^2 - 6k - 15 = \underline{2(2k^2 - 3k - 8) + 1}$

IS ODD SINCE $2k^2 - 3k - 8 \in \mathbb{Z}$.

THUS $n^2 - 5n - 11$ IS ODD FOR ALL $n \in \mathbb{Z}$.

④ PF SINCE $a \mid b$ AND $c \mid d$, $b = ka$ AND $d = \ell c$ FOR SOME $k, \ell \in \mathbb{Z}$.

THEN $\underline{ad + bc} = a(\ell c) + (ka)c = \underline{ac(\ell + k)}$ WHERE $\ell + k \in \mathbb{Z}$,

SO $ac \mid (ad + bc)$.

⑤ PF (BY CONTRADICTION)

SUPPOSE INSTEAD THAT $\sqrt{15}$ IS RATIONAL, SO $\sqrt{15} = \frac{m}{n}$ WHERE $m, n \in \mathbb{Z}$

WITH $n \neq 0$ AND $\frac{m}{n}$ IN REDUCED FORM.

THEN $15 = \frac{m^2}{n^2}$, SO $\underline{m^2 = 15n^2 = 5(3n^2)}$ WHERE $3n^2 \in \mathbb{Z}$.

THEREFORE $5 \mid m^2$ AND HENCE $5 \mid m$ (SINCE 5 IS PRIME),

SO $\underline{m = 5k}$ FOR SOME $k \in \mathbb{Z}$.

CONSEQUENTLY $m^2 = \underline{25k^2 = 15n^2}$, SO $\underline{5k^2 = 3n^2}$ WHERE $k^2 \in \mathbb{Z}$.

THUS $5 \mid 3n^2$ AND $5 \nmid 3$, SO $5 \mid n^2$ AND THEREFORE $5 \mid n$ (SINCE 5 IS PRIME).

THIS GIVES A CONTRADICTION, SINCE 5 IS A COMMON DIVISOR OF m AND n;

SO $\sqrt{15}$ IS IRRATIONAL.

REMARK #1 ON DISCUSSION SHEET 2 GIVES AN ALTERNATE APPROACH:

IF p IS A PRIME DIVISOR OF n, THEN $p \mid 15n^2 \Rightarrow p \mid m^2 \Rightarrow p \mid m$;

SO THIS WOULD GIVE A CONTRADICTION. THEREFORE n HAS NO PRIME

DIVISORS, SO $n = 1$ AND $15 = m^2$. SINCE $3^2 < 15 < 4^2$, THIS GIVES

A CONTRADICTION.

MATH 108   ANSWERS – QUIZ 1B

① PF (OF THE CONTRAPOSITIVE)

IF n IS EVEN, THEN $n = 2k$ FOR SOME $k \in \mathbb{Z}$,

THEN $(n-3)^2 = (2k-3)^2 = 4k^2 - 12k + 9 = 2(2k^2 - 6k + 4) + 1$

WHERE $2k^2 - 6k + 4 \in \mathbb{Z}$, SO $(n-3)^2$ IS ODD.

② A) $p \Rightarrow q \equiv \underline{\sim p} \vee q$ C) $\sim(p \wedge q) \equiv (\sim p) \underline{\vee} (\sim q)$ E) $p \Rightarrow (q \vee r) \equiv \left(p \wedge (\underline{\sim r})\right) \Rightarrow q$

B) $\sim(p \wedge q) \equiv p \underline{\Rightarrow} (\sim q)$ D) $\sim(p \Rightarrow q) \equiv p \underline{\wedge} (\sim q)$ F) $(p \vee q) \wedge (p \vee r) \equiv p \underline{\vee} (q \underline{\wedge} r)$

③ 1) IF n IS EVEN, THEN $n = 2k$ FOR SOME $k \in \mathbb{Z}$; SO

$\underline{n^2 - 7n - 9} = 4k^2 - 14k - 9 = \underline{2(2k^2 - 7k - 5) + 1}$ IS ODD SINCE $2k^2 - 7k - 5 \notin \mathbb{Z}$,

2) IF n IS ODD, THEN $n = 2k+1$ FOR SOME $k \in \mathbb{Z}$; SO

$\underline{n^2 - 7n - 9} = (2k+1)^2 - 7(2k+1) - 9 = 4k^2 - 10k - 15 = \underline{2(2k^2 - 5k - 8) + 1}$

IS ODD SINCE $2k^2 - 5k - 8 \in \mathbb{Z}$,

THUS $n^2 - 7n - 9$ IS ODD FOR ALL $n \in \mathbb{Z}$,

④ PF SINCE $c \mid m$ AND $c \mid n$, $m = kc$ AND $n = \ell c$ FOR SOME $k, \ell \in \mathbb{Z}$,

THEN $\underline{am + bn} = a(kc) + b(\ell c) = \underline{c(ak + b\ell)}$ WHERE $ak + b\ell \in \mathbb{Z}$,

SO $c \mid (am + bn)$.

⑤ PF (BY CONTRADICTION)

ASSUME INSTEAD THAT $\sqrt{35}$ IS RATIONAL, SO $\sqrt{35} = \dfrac{m}{n}$ WHERE $m, n \in \mathbb{Z}$

WITH $n \neq 0$ AND $\dfrac{m}{n}$ IN REDUCED FORM.

THEN $35 = \dfrac{m^2}{n^2}$, SO $\underline{m^2 = 35n^2 = 5(7n^2)}$ WHERE $7n^2 \in \mathbb{Z}$,

THEREFORE $5 \mid m^2$ AND HENCE $5 \mid m$ (SINCE 5 IS PRIME),

SO $\underline{m = 5k}$ FOR SOME $k \in \mathbb{Z}$,

CONSEQUENTLY $m^2 = \underline{25k^2 = 35n^2}$, SO $\underline{5k^2 = 7n^2}$ WHERE $k^2 \in \mathbb{Z}$.

THUS $5 \mid 7n^2$ AND $5 \nmid 7$, SO $5 \mid n^2$ AND THEREFORE $5 \mid n$ (SINCE 5 IS PRIME).

THIS GIVES A CONTRADICTION, SINCE 5 IS A COMMON DIVISOR OF m AND n;

SO $\sqrt{35}$ IS IRRATIONAL.

REMARK #1 ON DISCUSSION SHEET 2 GIVES AN ALTERNATE APPROACH!

IF p IS A PRIME DIVISOR OF n, THEN $p \mid 35n^2 \Rightarrow p \mid m^2 \Rightarrow p \mid m$;

SO THIS WOULD GIVE A CONTRADICTION. THEREFORE n HAS NO PRIME

DIVISORS, SO $n = 1$ AND $m^2 = 35$. SINCE $5^2 < 35 < 6^2$,

THIS GIVES A CONTRADICTION.