MATH 108 TEST 2 REVIEW PROBLEMS

(1) LET $A = \{a, b, c\}$. LIST THE ORDERED PAIRS IN A RELATION R ON A WHICH IS REFLEXIVE, NOT SYMMETRIC, AND NOT TRANSITIVE.

(2) DEFINE $f: \mathbb{R} - \{8\} \to \mathbb{R} - \{5\}$ BY $f(x) = \frac{5x}{x-8}$.

A) PROVE THAT f IS 1-1.

B) PROVE THAT f IS ONTO.

(3) DEFINE A RELATION S ON $\mathbb{R} - \{0\}$ BY $x S y$ IFF $\frac{x}{y} = 6^k$ FOR SOME $k \in \mathbb{Z}$.
PROVE THAT S IS AN EQUIVALENCE RELATION.

(4) GIVE A FORMULA FOR A BIJECTION f BETWEEN EACH OF THE FOLLOWING SETS.
(YOU DO NOT HAVE TO PROVE THAT f IS BIJECTIVE.)

A) $f: (0,1) \to (a,b)$ B) $f: \mathbb{Z} \to \mathbb{N}$

(5) LET $f: C \to D$ AND $g: D \to E$.

A) PROVE THAT IF $g \circ f$ IS 1-1, THEN f IS 1-1.

B) PROVE THAT IF f AND g ARE ONTO, THEN $g \circ f: C \to E$ IS ONTO.

(6) DEFINE A RELATION R ON $\mathbb{Z}$ BY $m R n$ IFF $8 \mid (5m + 3n)$.
SHOW THAT R IS A) SYMMETRIC B) TRANSITIVE.

(7) DEFINE $f: \mathbb{R} \to \mathbb{R}$ BY $f(x) = \begin{cases} \frac{1}{3-x}, & \text{IF } x < 3 \\ x^2 - 6x, & \text{IF } x \geq 3. \end{cases}$

A) SHOW THAT f IS NOT 1-1.

B) SHOW THAT f IS NOT ONTO.

(8) IF $f: (-2,2) \to \mathbb{R}$ IS DEFINED BY $f(x) = \frac{x}{x^2-4}$,
SHOW THAT f IS SURJECTIVE.

(9) DEFINE $f: \mathbb{R} \to \mathbb{R}$ BY $f(x) = \begin{cases} 15 - x^2, & \text{IF } x < 2 \\ x + 1, & \text{IF } x \geq 2. \end{cases}$

A) FIND $f((-1,7))$.

B) FIND $f^{-1}((6,11))$.

(10) FIND THE NUMBER OF EQUIVALENCE RELATIONS ON $A = \{1,2,3,4,5\}$,
AND JUSTIFY YOUR ANSWER.