① $A = \{a, b, c\}$ $R = \{(a,a), (b,b), (c,c), (a,b), (b,c)\}$, FOR EXAMPLE

② $f: \mathbb{R} - \{8\} \longrightarrow \mathbb{R} - \{5\}$ WITH $f(x) = \dfrac{5x}{x-8}$

A) IF $f(a) = f(b)$, THEN $\dfrac{5a}{a-8} = \dfrac{5b}{b-8} \Longrightarrow 5a(b-8) = 5b(a-8) \Longrightarrow 5ab - 40a = 5ab - 40b$
$\Longrightarrow -40a = -40b \Longrightarrow a = b$;
SO f IS 1-1.

B) LET $y \in \mathbb{R} - \{5\}$, AND LET $x = \dfrac{8y}{y-5}$.

THEN $x \in \mathbb{R} - \{8\}$ SINCE $8y \neq 8y - 40 \Longrightarrow 8y \neq 8(y-5) \Longrightarrow x = \dfrac{8y}{y-5} \neq 8$,

AND $f(x) = \dfrac{5\left(\frac{8y}{y-5}\right)}{\frac{8y}{y-5} - 8} = \dfrac{40y}{8y - 8(y-5)} = \dfrac{40y}{40} = y$;

SO $y \in \text{RANGE}(f)$ AND THEREFORE f IS ONTO.

③ $x \,S\, y$ IFF $\dfrac{x}{y} = 6^k$ FOR SOME $k \in \mathbb{Z}$, ON $\mathbb{R} - \{0\}$.

1) IF $x \in \mathbb{R} - \{0\}$, THEN $\dfrac{x}{x} = 1 = 6^0$ SO $x \,S\, x$ AND THEREFORE S IS REFLEXIVE.

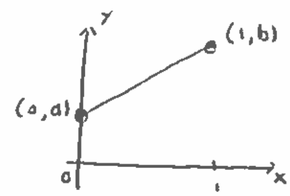
2) IF $x \,S\, y$, THEN $\dfrac{x}{y} = 6^k$ FOR SOME $k \in \mathbb{Z}$;
SO $\dfrac{y}{x} = 6^{-k}$ WHERE $-k \in \mathbb{Z}$ AND SO $y \,S\, x$.

THEREFORE S IS SYMMETRIC.

3) IF $x \,S\, y$ AND $y \,S\, z$, THEN $\dfrac{x}{y} = 6^k$ AND $\dfrac{y}{z} = 6^l$ FOR SOME $k, l \in \mathbb{Z}$;
SO $\dfrac{x}{z} = \dfrac{x}{y} \cdot \dfrac{y}{z} = 6^k \cdot 6^l = 6^{k+l}$ WHERE $k + l \in \mathbb{Z}$ AND HENCE $x \,S\, z$.

THEREFORE S IS TRANSITIVE.

④ A) $f: (0,1) \longrightarrow (a,b)$ LET $f(x) = (b-a)x + a$, FOR EXAMPLE.



B) $f: \mathbb{Z} \longrightarrow \mathbb{N}$ LET $f(n) = \begin{cases} 2n, & \text{IF } n > 0 \\ 1 - 2n, & \text{IF } n \leq 0 \end{cases}$, FOR EXAMPLE.

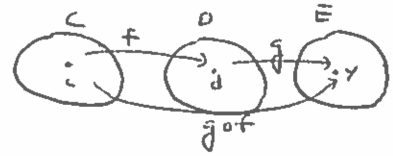
$$\begin{array}{lccccccccc} \mathbb{Z}: & 0 & 1 & -1 & 2 & -2 & 3 & -3 & \cdots \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ \mathbb{N}: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \end{array}$$

⑤ $f: C \rightarrow D$ AND $g: D \rightarrow E$

A) PROVE THAT IF $g \circ f$ IS 1-1, THEN f IS 1-1.

PF ASSUME THAT $g \circ f$ IS 1-1,
AND LET $f(a) = f(b)$.
THEN $g(f(a)) = g(f(b))$,
SO $(g \circ f)(a) = (g \circ f)(b)$
AND THEREFORE $a = b$ SINCE $g \circ f$ IS 1-1.
THUS f IS 1-1.

⑤ B) PROVE THAT IF f AND g ARE ONTO, THEN $g \circ f : C \to E$ IS ONTO.



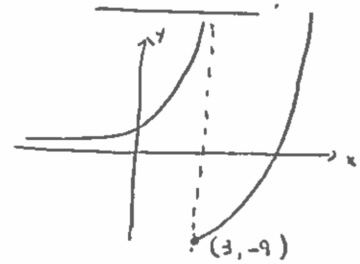
PF ASSUME THAT f AND g ARE ONTO, AND LET $y \in E$.
SINCE g IS ONTO, $y = g(d)$ FOR SOME $d \in D$; AND
SINCE f IS ONTO, $d = f(c)$ FOR SOME $c \in C$.
THEN $y = g(d) = g(f(c)) = (g \circ f)(c)$, SO $y \in \text{RANGE}(g \circ f)$
AND HENCE $g \circ f$ IS ONTO.

⑥ $m R n$ IFF $8 \mid (5m + 3n)$, ON $\mathbb{Z}$

A) IF mRn, THEN $8 \mid (5m+3n)$ SO $5m + 3n = 8k$ FOR SOME $k \in \mathbb{Z}$,
THEN $5n + 3m = (8m + 8n) - (5m + 3n) = 8m + 8n - 8k = 8(m+n-k)$ WHERE $m+n-k \in \mathbb{Z}$,
SO $8 \mid (5n + 3m)$ AND THEREFORE nRm. THUS R IS SYMMETRIC.

B) IF mRn AND nRp, THEN $8 \mid (5m+3n)$ AND $8 \mid (5n + 3p)$ SO
$5m + 3n = 8k$ AND $5n + 3p = 8\ell$ FOR SOME $k, \ell \in \mathbb{Z}$,
THEREFORE $5m + 3p = (5m+3n) + (5n+3p) - 8n = 8k + 8\ell - 8n = 8(k+\ell-n)$ WHERE $k+\ell-n \in \mathbb{Z}$,
SO $8 \mid (5m + 3p)$ AND THEREFORE mRp. THUS R IS TRANSITIVE.

⑦ $f : \mathbb{R} \to \mathbb{R}$ WITH $f(x) = \begin{cases} \dfrac{1}{3-x}, & \text{IF } x < 3 \\ x^2 - 6x, & \text{IF } x \geq 3. \end{cases}$



A) f IS NOT 1-1 SINCE
$f(7) = 7 = f\left(\frac{20}{7}\right)$, FOR EXAMPLE.
(OR $f(8) = 16 = f\left(\frac{47}{16}\right)$)

B) 1) IF $x < 3$, THEN $f(x) = \dfrac{1}{3-x} > 0$.
2) IF $x \geq 3$, THEN $f(x) = x^2 - 6x = (x^2 - 6x + 9) - 9 = (x-3)^2 - 9 \geq -9$.
THEREFORE RANGE $(f) \subseteq [-9, \infty)$, SO f IS NOT ONTO SINCE RANGE $(f) \neq \mathbb{R}$.

OR ASSUME THAT $-10 \in$ RANGE (f), SO $f(x) = -10$ FOR SOME $x \in \mathbb{R}$.
1) IF $x < 3$, THEN $\dfrac{1}{3-x} = -10$ SO $3 - x = -\frac{1}{10}$ AND $x = 3.1 > 3$,
WHICH GIVES A CONTRADICTION,
2) IF $x \geq 3$, THEN $x^2 - 6x = -10$ SO $x^2 - 6x + 9 = -10 + 9$ AND
THEREFORE $(x-3)^2 = -1$, WHICH GIVES A CONTRADICTION,
THEREFORE $-10 \notin$ RANGE(f), SO f IS NOT ONTO.

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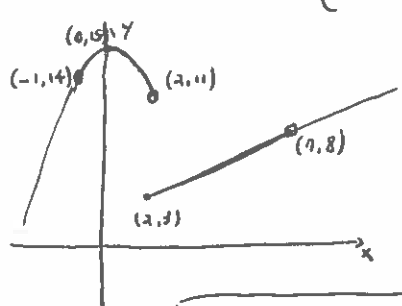
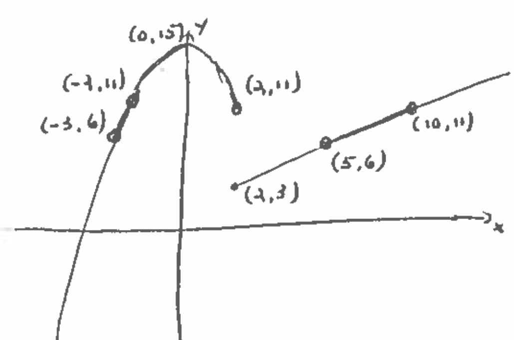
⑨ $f : \mathbb{R} \to \mathbb{R}$ WITH $f(x) = \begin{cases} 15 - x^2, & \text{IF } x < 2 \\ x + 1, & \text{IF } x \geq 2. \end{cases}$

A)

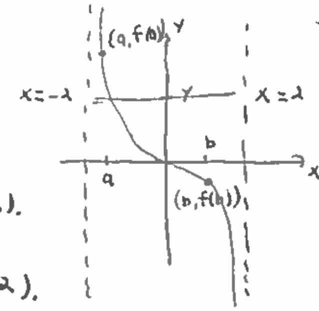


$f((-1,7)) = \boxed{[3,8) \cup (11,15]}$

B)



$f^{-1}((6,11)) = \boxed{(-3,-2) \cup (5,10)}$



(8) $f: (-2,2) \to \mathbb{R}$ defined by $f(x) = \dfrac{x}{x^2-4}$ is surjective.

PF Let $Y \in \mathbb{R}$.

1) Since $\lim_{x \to -2^+} f(x) = \infty$, $f(a) > Y$ for some $a \in (-2,2)$.

2) Since $\lim_{x \to 2^-} f(x) = -\infty$, $f(b) < Y$ for some $b \in (a,2)$.

Since f is continuous on $[a,b]$, by the Intermediate Value Th.

There is an $x \in (a,b)$ with $f(x) = Y$.

Therefore $Y \in \text{Range}(f)$, so f is surjective.

(10) The number of equivalence relations on $A = \{1,2,3,4,5\}$

is equal to the number of partitions of A,

and we can classify the partitions by the sizes of the subsets they contain:

1) 5 — Number: 1

2) 4/1 — Number: 5

3) 3/2 — Number: $\binom{5}{2} = \frac{5\cdot4}{2} = 10$

4) 3/1/1 — Number: $\frac{5\cdot4}{2} = 10$

5) 2/2/1 — Number: $5\cdot3 = 15$

6) 2/1/1/1 — Number: $\binom{5}{2} = \frac{5\cdot4}{2} = 10$

7) 1/1/1/1/1 — Number: 1

This gives a total of $\boxed{52}$ partitions.

REMARK This can also be found using the Bell Triangle,
which is analogous to Pascal's Triangle:

1
1 2
2 3 5
5 7 10 15
15 20 27 37 52
52 67 87 114 151 203