MATH 108   **Discussion Sheet 2**

(1) Complete the proof of the following theorem (which shows that $\sqrt{5}$, $\sqrt[3]{2}$, $\sqrt[5]{33}$, $\sqrt[4]{12}$ are all irrational):

Th If n and k are positive integers and n is not a perfect kth power [so $\sim(\exists m \in \mathbb{N})(m^k = n)$], then $\sqrt[k]{n}$ is irrational.

Pf (by contradiction)
Suppose instead that $\sqrt[k]{n}$ is rational, so $\sqrt[k]{n} = \frac{c}{d}$ where $c \in \mathbb{Z}$, $d \in \mathbb{N}$, and c and d have no common prime factor. Then $n = \frac{c^k}{d^k}$, so $c^k = n d^k$.
If p is a prime divisor of d, then $p \mid n d^k$ so $p \mid c^k$ ...

(2) Let P_1, P_2, and q be propositions.
If $P_1 \Rightarrow P_2$ is true, which of the following must be true?

a) $(P_1 \Rightarrow q) \Rightarrow (P_2 \Rightarrow q)$

b) $(P_2 \Rightarrow q) \Rightarrow (P_1 \Rightarrow q)$

(3) Prove that $x^2 + y^2 + z^2 \geq xy + yz + xz$ for all $x, y, z \in \mathbb{R}$.

(4) Let A, B, C be sets. Show that

a) $(A - B) \cup (A - C) = A - B \cap C$,

b) $A - (B - C) = (A \cap C) \cup (A - B)$.

(5) Use induction to prove that

A) $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$ for all $n \in \mathbb{N}$,

B) $3^n > n^2$ for all $n \in \mathbb{N}$.

C) $1 + 5 + 9 + \cdots + (4n-3) = 2n^2 - n$ for all $n \in \mathbb{N}$.