MATH 108 DISCUSSION SHEET 3

DEF LET R BE A RELATION ON A SET A. WE SAY THAT R IS

a) REFLEXIVE IFF aRa $\forall a \in A$.

b) SYMMETRIC IFF $aRb \Rightarrow bRa$ $\forall a, b \in A$.

c) TRANSITIVE IFF aRb AND $bRc \Rightarrow aRc$ $\forall a, b, c \in A$.

DEF A RELATION R ON A SET A IS AN EQUIVALENCE RELATION IFF R IS REFLEXIVE, SYMMETRIC, AND TRANSITIVE.

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① FOR EACH OF THE FOLLOWING RELATIONS, DETERMINE WHETHER THE RELATION IS a) REFLEXIVE b) SYMMETRIC c) TRANSITIVE:

A) $m \mid n$ ON $\mathbb{N}$

B) $x \leq y$ ON $\mathbb{R}$

C) $a \neq b$ ON $\mathbb{Z}$

D) $x > y$ ON $\mathbb{R}$

E) aRb IFF $ab \geq 0$ ON $\mathbb{Z}$

② LET $A = \{a, b, c, d\}$. GIVE AN EXAMPLE OF A RELATION R ON A WHICH IS

a) REFLEXIVE AND SYMMETRIC BUT NOT TRANSITIVE.

b) REFLEXIVE AND TRANSITIVE BUT NOT SYMMETRIC.

c) SYMMETRIC AND TRANSITIVE BUT NOT REFLEXIVE.

③ LET $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, AND LET $X_1 = \{1, 2, 3, 4\}$, $X_2 = \{3, 4, 5, 6\}$, $X_3 = \{4, 6, 8\}$, AND $X_4 = \{1, 3, 5, 8\}$.

DEFINE A RELATION R ON A BY mRn IFF $m \in X_i$ AND $n \in X_i$ FOR SOME i, $1 \leq i \leq 4$.

DETERMINE IF R IS a) REFLEXIVE b) SYMMETRIC c) TRANSITIVE.

④ LET f BE A FUNCTION FROM A SET X TO A SET Y.

IF S IS A RELATION ON Y, DEFINE A RELATION R ON X BY aRb IFF $f(a) \, S \, f(b)$.

WHICH OF THE FOLLOWING STATEMENTS IS TRUE?

A) IF S IS REFLEXIVE THEN R IS REFLEXIVE.

B) IF S IS SYMMETRIC THEN R IS SYMMETRIC.

C) IF S IS TRANSITIVE THEN R IS TRANSITIVE.

⑤ DEFINE A RELATION R ON $\mathbb{Z}$ BY mRn IFF $5 \mid (m-n)$.

SHOW THAT R IS AN EQUIVALENCE RELATION ON $\mathbb{Z}$.