MATH 108 DISCUSSION SHEET 5

1. A) SHOW THAT IF $f: A \to \mathbb{N}$ IS 1-1, THEN A IS COUNTABLE.

   B) SHOW THAT IF $f: \mathbb{R} \to \mathbb{N}$, THEN f IS NOT 1-1.

2. LET B BE A SUBSET OF A.

   A) SHOW THAT IF B IS FINITE AND A IS DENUMERABLE, THEN $A - B$ IS DENUMERABLE.

   B) SHOW THAT IF B IS COUNTABLE AND A IS UNCOUNTABLE, THEN $A - B$ IS UNCOUNTABLE.

3. LET S BE THE SET OF NUMBERS IN $(0,1)$ WHOSE DECIMAL EXPANSIONS CONTAIN NO DIGITS OTHER THAN 4 AND 8.
   PROVE OR DISPROVE THAT S IS COUNTABLE.

4. DEFINE $f: (0,1) \times (0,1) \to (0,1)$ AS FOLLOWS:
   IF $X, Y \in (0,1)$ WITH $X = .X_1 X_2 X_3 X_4 \cdots$ AND $Y = .Y_1 Y_2 Y_3 Y_4 \cdots$ (IN NORMALIZED FORM), THEN $f(X,Y) = .X_1 Y_1 X_2 Y_2 X_3 Y_3 \cdots$.
   PROVE OR DISPROVE THAT

   A) f IS INJECTIVE.

   B) f IS SURJECTIVE.

5. LET $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ BE DEFINED BY $f(m,n) = \dfrac{(m+n-1)(m+n-2)}{2} + m$.

   A) PROVE THAT f IS INJECTIVE.

   B) PROVE THAT f IS SURJECTIVE.

6. DEFINE $f: [0,1] \to (0,1)$ BY

   a) $f(0) = \frac{1}{3}$ AND $f(1) = \frac{2}{3}$,

   b) $f\left(\frac{1}{m}\right) = \frac{1}{m+1}$ FOR $m \geq 3$,

   c) $f\left(\frac{m-1}{m}\right) = \frac{m}{m+1}$ FOR $m \geq 3$, AND

   d) $f(x) = x$ OTHERWISE.

   DETERMINE WHETHER OR NOT f IS A BIJECTION.