MATH 108

TEST 1 Review PROBLEMS

(1) Fill in the blanks to provide equivalent propositional forms:

A) $p \Rightarrow q \equiv$ ___ $\vee q$

B) $\sim(p \vee q) \equiv (\sim p)$ ___ $(\sim q)$

C) $\sim(p \wedge q) \equiv p$ ___ $(\sim q)$

D) $p \Rightarrow (q \vee r) \equiv (p \wedge$ ___ $) \Rightarrow r$

(2) Let $a, b, c \in \mathbb{Z}$. Prove that if $a \mid b$ and $a \mid c$, then $a \mid (b+c)$.

(3) Let $n \in \mathbb{Z}$. Prove that if $(n+5)^2$ is odd, then n is even.

(4) Define $\underline{\lim_{x \to a} f(x) = L}$ iff $(\forall \varepsilon)\{\varepsilon > 0 \Rightarrow [(\exists \delta)(\delta > 0 \wedge (\forall x)(0 < |x-a| < \delta \Rightarrow |f(x) - L| < \varepsilon))]\}$.

Find a symbolic denial of $\lim_{x \to a} f(x) = L$, moving the negation as far to the right as possible.

(5) Let $x, y \in \mathbb{R}$. Prove that if $x \in \mathbb{Q}$ and $y \notin \mathbb{Q}$, then $x + y \notin \mathbb{Q}$.

(6) Let $A_n = \left[2 + \frac{4}{n},\ 8 + \frac{12}{n}\right) \ \forall n \in \mathbb{N}$. Find the following:

a) $\bigcap_{n \in \mathbb{N}} A_n =$ ______

b) $\bigcup_{n \in \mathbb{N}} A_n =$ ______

(7) Let A, B, C, and D be sets. Prove that if $A \cup B \subseteq C \cup D$, $A \cap B = \emptyset$, and $D \subseteq B$, then $A \subseteq C$.

(8) Prove that $3^n > n^3$ for all integers $n \geq 4$ using the PMI.

(9) Prove that $\sqrt[5]{3}$ is irrational.

(You may use the result that if p is prime and $p \mid m^5$, then $p \mid m$, for any $m \in \mathbb{N}$.)

(10) Prove that any integer $n \geq 24$ can be written in the form

$n = 4a + 9b$ for some integers $a, b \geq 0$ using the PSI (or PMI).

(11) Use the WOP to prove the PMI.