4.2 – (2d) $15x \equiv 9 \pmod{25}$ $(15,25) = 5$ AND $5 \nmid 9$, SO THERE IS $\boxed{\text{NO SOLUTION}}$.

(5) LET T BE THE NUMBER OF HOURS IN AN ORBIT, SO $1 \le T < 24$ AND $11T \equiv 17 \pmod{24}$.

THEN $11T - 24Y = 17$ FOR SOME $Y \in \mathbb{Z}$. USING THE EUCLIDEAN ALGORITHM,

				s_i	t_i
24	11	2	2	1	0
11	2	5	1	0	1
2	1	2	0	1	-2
1				-5	11

$-5(24) + (11)(11) = 1$, SO

$-85(24) + 187(11) = 17$. TAKING $T_0 = 187$ AND $Y_0 = 85$,

$T = 187 + 24u$, $Y = 85 + 11u$, $u \in \mathbb{Z}$

SO $T \equiv 187 \equiv \boxed{19} \pmod{24}$, SO AN ORBIT LASTS $\boxed{19 \text{ HRS}}$

(10) a) IF $1 \le a \le 14$, a HAS AN INVERSE MOD 14 IFF $(a,14) = 1$, SO $a = \boxed{1, 3, 5, 9, 11, \text{ OR } 13}$

b) $\bar{1} = \boxed{1}$, $\bar{3} = \boxed{5}$, $\bar{5} = \boxed{3}$, $\bar{9} = \overline{(-5)} \equiv -3 \equiv \boxed{11}$, $\overline{11} = \boxed{9}$, $\overline{13} \equiv \overline{(-1)} \equiv -1 \equiv \boxed{13} \pmod{14}$

4.3 – (2) $x \equiv 1 \pmod 2$, $x \equiv 1 \pmod 5$, $x \equiv 0 \pmod 3$

$M_1 = 15 \equiv 1 \pmod 2$ $M_2 = 6 \equiv 1 \pmod 5$ $M_3 = 10 \equiv 1 \pmod 3$

LET $Y_1 = 1$, $Y_2 = 1$, $Y_3 = 1$ TO GET

$x = 1 \cdot 15 \cdot 1 + 1 \cdot 6 \cdot 1 + 0 \cdot 10 \cdot 1 = \boxed{21}$. (THEREFORE $x = 21 + 30T$, $T \in \mathbb{Z}$ GIVES ALL SUCH INTEGERS)

(4a) $x \equiv 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

$M_1 = 17 \equiv 6 \pmod{11}$ $M_2 = 11 \equiv -6 \pmod{17}$

LET $Y_1 = 2$ AND $Y_2 = -3$ TO GET $x = 4 \cdot 17 \cdot 2 + 3(11)(-3) = 37$,

SO $\boxed{x \equiv 37 \pmod{187}}$

OR: $x \equiv 3 \pmod{17}$, SO $x = 3 + 17T$. THEN $x \equiv 4 \pmod{11}$ GIVES $3 + 17T \equiv 4 \pmod{11}$, SO $6T \equiv 1 \pmod{11}$. THEN $6T \equiv 12 \pmod{11}$, SO $T \equiv 2 \pmod{11}$ AND THEREFORE $T = 2 + 11u$. THUS $x = 3 + 17(2 + 11u) = 37 + 187u$, SO $\boxed{x \equiv 37 \pmod{187}}$

(4b) $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$

$M_1 = 15 \equiv 1 \pmod 2$ $M_2 = 10 \equiv 1 \pmod 3$ $M_3 = 6 \equiv 1 \pmod 5$

LET $Y_1 = 1$, $Y_2 = 1$, $Y_3 = 1$ TO GET

$x = 1 \cdot 15 \cdot 1 + 2 \cdot 10 \cdot 1 + 3 \cdot 6 \cdot 1 = 53$, SO $\boxed{x \equiv 23 \pmod{30}}$

OR: $x \equiv 3 \pmod 5$, SO $x = 3 + 5T$. THEN $x \equiv 2 \pmod 3$ GIVES $3 + 5T \equiv 2 \pmod 3$, SO $2T \equiv -1 \equiv 2 \pmod 3$ AND $T \equiv 1 \pmod 3$. THEN $T = 1 + 3u$, SO $x = 3 + 5(1 + 3u) = 8 + 15u$. SINCE $x \equiv 1 \pmod 2$, $8 + 15u \equiv 1 \pmod 2$ SO $u \equiv 1 \pmod 2$. THEN $u = 2v + 1$ GIVES $x = 8 + 15(2v + 1) = 23 + 30v$, SO $\boxed{x \equiv 23 \pmod{30}}$

4.3-(7) Let X be the number of bananas, so $X \equiv 6 \pmod{11}$ and $X \equiv 0 \pmod{17}$, with $X \geq 11(2)+6=28$.
Then $X = 17T$, so $17T \equiv 6 \pmod{11}$ gives $6T \equiv 6 \pmod{11}$ and hence $T \equiv 1 \pmod{11}$.
Then $T = 1+11u$ gives $X = 17(1+11u) = 17+187u$. Since $X \geq 28$, the smallest possible value of X is given by $X = 17+187 = \boxed{204}$ bananas.

(10) $X \equiv 9 \pmod{10}$, $X \equiv 9 \pmod{11}$, $X \equiv 0 \pmod{13}$

Since $X \equiv 0 \pmod{13}$, $X = 13T$; so $X \equiv 9 \pmod{11}$ gives $13T \equiv 9 \pmod{11}$ or $2T \equiv 20 \pmod{11}$.
Then $T \equiv 10 \equiv -1 \pmod{11}$, so $T = -1+11u$. Then $X = 13(-1+11u) = -13+143u$, and $X \equiv 9 \pmod{10}$ implies $-13+143u \equiv 9 \pmod{10}$ and therefore $3u \equiv 22 \equiv 12 \pmod{10}$.
Then $u \equiv 4 \pmod{10}$, so $u = 4+10v$ gives $X = -13+143(4+10v) = 559+1430v$.
Taking $v=0$ gives $\boxed{X = 559}$ as one such integer.

OR $X \equiv 9 \pmod{10}$, $X \equiv 9 \pmod{11}$, $X \equiv 0 \pmod{13}$

$M_1 = 143 \equiv 3 \pmod{10}$ $\quad M_2 = 130 \equiv -2 \pmod{11}$ $\quad M_3 = 110 \equiv 6 \pmod{13}$

Let $y_1 = 7$ $\quad y_2 = -6$ $\quad y_3 = 11$ to get

$x = 9(143)(7) + 9(130)(-6) + 0(110)(11) = \boxed{1989}$ as a solution.

More generally, $X \equiv 1989 \equiv 559 \pmod{1430}$.

(16a) $X \equiv 4 \pmod{6}$, $X \equiv 13 \pmod{15}$ $\quad (6,15)=3$, and $4 \equiv 13 \pmod{3}$; so there is a solution.

Since $X \equiv 13 \pmod{15}$, $X = 13+15T$ for some $T \in \mathbb{Z}$. Then $X \equiv 4 \pmod{6}$ gives $13+15T \equiv 4 \pmod{6}$, so $15T \equiv -9 \pmod{6}$ and hence $3T \equiv 3 \pmod{6}$. Then $T \equiv 1 \pmod{2}$, so $T = 1+2u$ for some $u \in \mathbb{Z}$. Then $X = 13+15(1+2u) = 28+30u$, so $\boxed{X \equiv 28 \pmod{30}}$

OR $X \equiv 4 \pmod{6}$, $X \equiv 13 \pmod{15}$ gives

$X \equiv 4 \pmod{2}$, $X \equiv 13 \pmod{5}$ so $X \equiv 0 \pmod{2}$, $X \equiv 1 \pmod{3}$
$X \equiv 4 \pmod{3}$, $X \equiv 13 \pmod{5}$ $\quad X \equiv 1 \pmod{3}$, $X \equiv 3 \pmod{5}$

Then $X \equiv 0 \pmod{2}$, $X \equiv 1 \pmod{3}$, $X \equiv 3 \pmod{5}$

$M_1 = 15 \equiv 1 \pmod{2}$ $\quad M_2 = 10 \equiv 1 \pmod{3}$ $\quad M_3 = 6 \equiv 1 \pmod{5}$

Let $y_1 = 1$, $\quad y_2 = 1$, $\quad y_3 = 1$ to get

$x = 0\cdot15\cdot1 + 1\cdot10\cdot1 + 3\cdot6\cdot1 = 28$; so $\boxed{X \equiv 28 \pmod{30}}$

(16b) $X \equiv 7 \pmod{10}$, $X \equiv 4 \pmod{15}$

Since $(10,15)=5$ and $7 \not\equiv 4 \pmod{5}$, there is $\boxed{\text{no solution}}$

4.3 (20a) $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{15}$

SINCE $5 \equiv 3 \pmod 2$, $5 \equiv 8 \pmod 3$, AND $3 \equiv 8 \pmod 5$, THERE IS A SOLUTION.

SINCE $x \equiv 8 \pmod{15}$, $x = 8 + 15t$ FOR SOME $t \in \mathbb{Z}$.

SINCE $x \equiv 3 \pmod{10}$, $8 + 15t \equiv 3 \pmod{10}$ SO $15t \equiv -5 \pmod{10}$ AND THUS $5t \equiv 5 \pmod{10}$.
THEN $t \equiv 1 \pmod 2$, SO $t = 1 + 2u$ FOR SOME $u \in \mathbb{Z}$, AND $x = 8 + 15(1+2u) = 23 + 30u$.
SINCE $x \equiv 5 \pmod 6$, $23 + 30u \equiv 5 \pmod 6$ WHICH GIVES $0 \equiv 0 \pmod 6$.
THEREFORE $\boxed{x \equiv 23 \pmod{30}}$

OR $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{15}$ GIVES

$x \equiv 5 \pmod 2$, $x \equiv 3 \pmod 2$, $x \equiv 8 \pmod 3$

$x \equiv 5 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 8 \pmod 5$

SO $x \equiv 1 \pmod 2$, ~~$x \equiv 1 \pmod 2$~~, $x \equiv 2 \pmod 3$

$x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, ~~$x \equiv 3 \pmod 5$~~

THEREFORE $x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$;

SO PROCEEDING AS IN 4b GIVES $\boxed{x \equiv 23 \pmod{30}}$

(20c) $x \equiv 2 \pmod 9$, $x \equiv 8 \pmod{15}$, $x \equiv 10 \pmod{25}$

SINCE $8 \not\equiv 10 \pmod 5$, AND $5 = (15, 25)$, THERE IS $\boxed{\text{NO SOLUTION}}$.

6.1 - (2) $12! = 1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10\cdot11\cdot12$

$\equiv 1\cdot(2\cdot7)\cdot(3\cdot9)\cdot(4\cdot10)\cdot(5\cdot8)\cdot(6\cdot11)\cdot12 \equiv 1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot12 \equiv 12 \equiv -1 \pmod{13}$,

so $13 \mid (12!+1)$.

(3) $18! \equiv -1 \pmod{19}$, so $16!\cdot17\cdot18 \equiv -1 \pmod{19}$ gives $16!(-2)(-1) \equiv 18 \pmod{19}$

and therefore $16! \equiv \boxed{9} \pmod{19}$

(7) $21^2 - 437 = 4$, so $437 = 21^2 - 2^2 = 19\cdot23$. Then using Wilson's Theorem,

$18! \equiv -1 \pmod{19}$, and $22! \equiv -1 \pmod{23}$ gives $18!\cdot19\cdot20\cdot21\cdot22 \equiv -1 \pmod{23}$

so $18!\cdot(-4)(-3)(-2)(-1) \equiv -1 \pmod{23}$. Then $18!\cdot24 \equiv -1 \pmod{23}$, so $18!\cdot1 \equiv -1 \pmod{23}$

and therefore $18! \equiv -1 \pmod{23}$.

Since $18! \equiv -1 \pmod{19}$ and $18! \equiv -1 \pmod{23}$, $18! \equiv -1 \pmod{437}$ so $18! \equiv \boxed{436} \pmod{437}$

(8) By Wilson's Theorem, $40! \equiv -1 \pmod{41}$ and $42! \equiv -1 \pmod{43}$.

Then $40!\cdot41\cdot42 \equiv -1 \pmod{43}$, so $40!(-2)(-1) \equiv 42 \pmod{43}$ gives $40! \equiv 21 \pmod{43}$.

Therefore $40!$ is a solution of the system

$x \equiv -1 \pmod{41}$ and $x \equiv 21 \pmod{43}$:

Since $x \equiv -1 \pmod{41}$, $x = -1 + 41t$ for some $t \in \mathbb{Z}$; so $x \equiv 21 \pmod{43}$

implies $-1 + 41t \equiv 21 \pmod{43}$. Then $-2t \equiv 22 \pmod{43}$, so $t \equiv -11 \equiv 32 \pmod{43}$.

Then $t = 32 + 43u$ for some $u \in \mathbb{Z}$, and $x = -1 + 41(32 + 43u) = 1311 + 1763u$.

Thus $40! \equiv \boxed{1311} \pmod{1763}$

(10) By Fermat's Theorem, $6^{10} \equiv 1 \pmod{11}$;

so $6^{2000} = (6^{10})^{200} \equiv 1^{200} \equiv \boxed{1} \pmod{11}$.

(12) Since $2^{16} \equiv 1 \pmod{17}$ by Fermat's Theorem,

$2^{1{,}000{,}000} = (2^{16})^{62{,}500} \equiv 1^{62{,}500} \equiv \boxed{1} \pmod{17}$