MATH 145

CH. 2 – SOLUTIONS

1 i) NO RESTRICTIONS: $5\cdot5\cdot5\cdot5 = \boxed{5^4}$

ii) DISTINCT DIGITS: $\boxed{5\cdot4\cdot3\cdot2}$

iii) EVEN: $\boxed{5\cdot5\cdot5\cdot2}$

iv) DISTINCT DIGITS AND EVEN: $\boxed{4\cdot3\cdot2\cdot2}$ (2 CHOICES FOR LAST DIGIT, THEN 4 CHOICES FOR FIRST DIGIT, 3 FOR THE 2ND DIGIT, AND 2 FOR THE 3RD DIGIT)

2 1) ORDER THE SUITS: $4!$ WAYS

2) ORDER THE CARDS IN EACH SUIT: $13!$ WAYS

ANSWER: $4!\cdot13!\cdot13!\cdot13!\cdot13! = \boxed{4!\cdot(13!)^4}$

6 SINCE THE DIGITS ARE DISTINCT AND THERE ARE ONLY 8 DIGITS AVAILABLE, THE INTEGERS CAN HAVE AT MOST 8 DIGITS:

a) 8 DIGITS: $7\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 35{,}280$

b) 7 DIGITS: $7\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2 = 35{,}280$

c) 6 DIGITS: $7\cdot7\cdot6\cdot5\cdot4\cdot3 = 17{,}640$

d) 5 DIGITS: $7\cdot7\cdot6\cdot5\cdot4 = 5{,}880$

e) 4 DIGITS: $\underbrace{1\cdot4\cdot6\cdot5}_{\text{(LEADING DIGIT 5)}} + \underbrace{3\cdot7\cdot6\cdot5}_{\text{(LEADING DIGIT 6,8,9)}} = 750$

USING THE ADDITION PRINCIPLE, THERE ARE $\boxed{94{,}830}$ SUCH NUMBERS.

9 a) FIRST SEAT A. 1) CHOICES FOR B: 12 2) WAYS TO SEAT OTHER 13 PEOPLE: $13!$

ANSWER: $\boxed{12\cdot13!}$

OR, BY THE SUBTRACTION PRINCIPLE, $\boxed{\underbrace{14!}_{\text{(TOTAL)}} - \underbrace{2\cdot13!}_{\text{(A AND B TOGETHER)}}}$

b) FIRST SEAT A. 1) CHOICES FOR B: 13 2) WAYS TO SEAT OTHER 13 PEOPLE: $13!$

ANSWER: $\boxed{13\cdot13!}$

OR, BY THE SUBTRACTION PRINCIPLE, $\boxed{\underbrace{14!}_{\text{(TOTAL)}} - \underbrace{13!}_{\text{(B ON A's RIGHT)}}}$

10 a) ANSWER: $\boxed{\underbrace{\binom{22}{5}}_{\text{(TOTAL)}} - \underbrace{\binom{10}{5}}_{\text{(0 WOMEN)}} - \underbrace{\binom{10}{4}\binom{12}{1}}_{\text{(1 WOMAN)}}}$ OR $\boxed{\underbrace{\binom{10}{3}\binom{12}{2}}_{(2W)} + \underbrace{\binom{10}{2}\binom{12}{3}}_{(3W)} + \underbrace{\binom{10}{1}\binom{12}{4}}_{(4W)} + \underbrace{\binom{12}{5}}_{(5W)}}$

14 1) SEAT 5 WHO SIT IN FRONT: $8\cdot7\cdot6\cdot5\cdot4$ CHOICES

2) SEAT 4 WHO SIT IN BACK: $8\cdot7\cdot6\cdot5$ CHOICES

3) SEAT 5 REMAINING STUDENTS IN 7 REMAINING SEATS: $7\cdot6\cdot5\cdot4\cdot3$ CHOICES

ANSWER: $\boxed{(8\cdot7\cdot6\cdot5\cdot4)\cdot(8\cdot7\cdot6\cdot5)\cdot(7\cdot6\cdot5\cdot4\cdot3)}$

15 a) THERE ARE $\boxed{P(20,15) = \frac{20!}{5!}}$ WAYS FOR THE MEN TO CHOOSE THEIR PARTNERS.

b) 1) CHOOSE THE MEN: $\binom{15}{10}$ WAYS 2) CHOOSE THE WOMEN: $\binom{20}{10}$ WAYS

3) PAIR THEM UP: $10!$ WAYS ANSWER: $\boxed{\binom{15}{10}\binom{20}{10}\cdot10!}$

MATH 145 CH. 2 – SOLUTIONS

7. 1) SEAT THE MEN IN A CIRCLE: $3!$ WAYS

2) SEAT THE WOMEN IN THE 8 POSITIONS BETWEEN THE MEN: $8!$ WAYS

ANSWER: $\boxed{3!\,8!}$

OR 1) SEAT THE WOMEN IN A CIRCLE: $7!$ WAYS

2) CHOOSE THE 4 GAPS FOR THE MEN: 2 WAYS

3) SEAT THE MEN IN THEIR POSITIONS: $4!$ WAYS

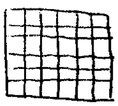
ANSWER: $\boxed{7!\cdot 2\cdot 4!}$

10. b) WE MUST SUBTRACT THE NUMBER OF COMMITTEES WHICH HAVE TOM AND BETTY AND AT LEAST 2 WOMEN:

THERE ARE $\binom{20}{3}$ COMMITTEES WITH TOM AND BETTY, AND $\binom{9}{3}$ OF THEM HAVE 4 MEN AND BETTY;

SO THE ANSWER IS $\boxed{\binom{22}{5}-\binom{10}{5}-\binom{10}{4}\binom{12}{1}-\left[\binom{20}{3}-\binom{9}{3}\right]}$

$= \boxed{\binom{22}{5}-\binom{10}{5}-\binom{10}{4}\binom{12}{1}-\binom{20}{3}+\binom{9}{3}}$

17. 

a) ANSWER: $\boxed{6!}$ SINCE THE ROOK IN ROW 1 HAS 6 CHOICES FOR ITS COLUMN, AND THEN THE ROOK IN ROW 2 HAS 5 CHOICES FOR ITS COLUMN, ETC.

b) 1) ARRANGE THE ROOKS: $6!$ CHOICES

2) SELECT 2 ROOKS TO COLOR RED: $\binom{6}{2}$ CHOICES

ANSWER: $\boxed{6!\cdot\binom{6}{2}}$

21. a) ADDRESSES LET $T = \{1\cdot A, 1\cdot R, 2\cdot D, 2\cdot E, 3\cdot S\}$.

NUMBER OF PERMUTATIONS OF T:

$$\boxed{\underset{S\text{'s}}{\binom{9}{3}}\underset{E\text{'s}}{\binom{6}{2}}\underset{D\text{'s}}{\binom{4}{2}}\underset{R}{\binom{2}{1}} = \frac{9!}{3!\,2!\,2!}}$$

b) THE NUMBER OF 8-PERMUTATIONS IS THE SAME, SINCE EVERY 8-PERMUTATION OF T CAN BE MATCHED WITH A PERMUTATION OF T BY PLACING THE MISSING LETTER AT THE END.

(FOR EXAMPLE, DARESESD $\longleftrightarrow$ DARESESD|S)

28. a) A ROUTE CORRESPONDS TO A PERMUTATION OF $\{9\cdot E, 8\cdot N\}$,

SO THERE ARE $\boxed{\binom{17}{9}=\binom{17}{8}}$ POSSIBLE ROUTES.

b) WE CAN FIND THE NUMBER OF ROUTES WHICH USE THE BLOCK WHICH IS UNDER WATER BY MULTIPLYING THE NUMBER OF ROUTES FROM $(0,0)$ TO $(4,3)$ BY THE NUMBER OF ROUTES FROM $(4,4)$ TO $(9,8)$;

USING THE SUBTRACTION PRINCIPLE,

THERE ARE $\boxed{\binom{17}{9}-\binom{7}{4}\binom{9}{5}}$ ROUTES STILL AVAILABLE.