MATH 145 SOLUTIONS - GREEN SHEET

① A) $val(F) = 10 + 2 = 11 + 1 = \underline{12}$

B) $cap(C) = C_{sb} + C_{ad} + C_{ct} = 11 + 1 + 11 = \underline{26}$

C) $\sum_{u \in S,\, v \in T} (f_{uv} - f_{vu}) = f_{sb} + f_{ad} + f_{ct} - f_{bc} = 2 + 0 + 11 - 1 = \underline{12} = val(F)$

② A) $val(F) = 2 + 4 = \underline{6}$

B) $cap(C) = C_{ac} + C_{ae} + C_{be} + C_{bd} = 3 + 1 + 4 + 3 = \underline{11}$

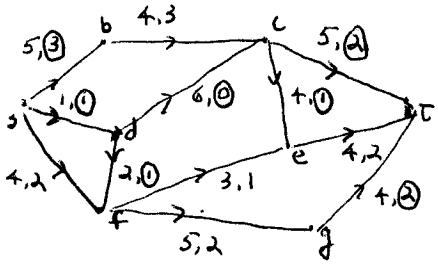
C) $\sum_{u \in S,\, v \in T} (f_{uv} - f_{vu}) = f_{ac} + f_{ae} + f_{be} + f_{bd} = 2 + 0 + 1 + 3 = \underline{6} = val(F)$

③ A) $val(F) = 11 + 10 = 9 + 12 = \underline{21}$

B) $cap(C) = C_{ac} + C_{be} + C_{dt} = 3 + 9 + 9 = \underline{21}$

C) $\sum_{u \in S,\, v \in T} (f_{uv} - f_{vu}) = f_{ac} + f_{be} + f_{dt} - f_{cd} = 3 + 9 + 9 - 0 = \underline{21} = val(F)$

④



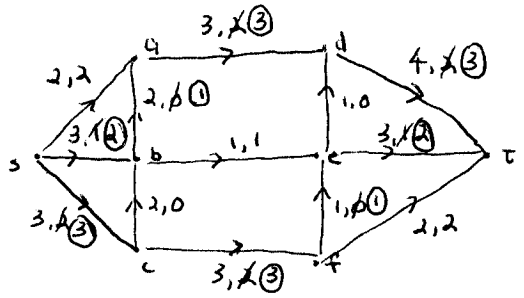
A) $val(F) = 3 + 1 + 2 = 2 + 2 + 2 = \underline{6}$

B) $cap(C) = C_{bc} + C_{dc} + C_{fe} + C_{fg} = 4 + 6 + 3 + 5 = \underline{18}$

(since $f_{df} = 1$, $f_{sd} \geq 1$ so $f_{sd} = 1$ and $f_{dc} = 0$)

MATH 145 SOLUTIONS – REC SHEET

(1)



A) $sbadT$ AND $scfeT$ ARE FLOW-AUGMENTING PATHS.

B) 1) USING $sbadT$: $\Delta = 1$

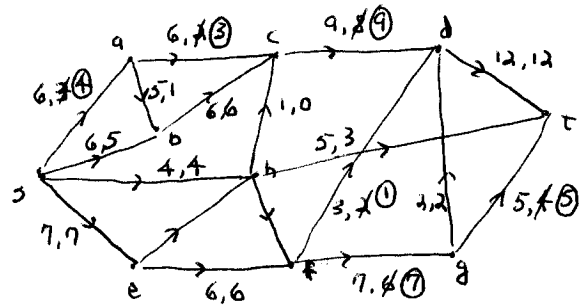
2) USING $scfeT$: $\Delta = 1$

LET $C = \{s, T\}$ WHERE $S = \{s, a, b\}$

AND $T = \{c, d, e, f, T\}$

$cap(C) = C_{sc} + C_{be} + C_{ad} = 3 + 1 + 3 = 7 = val(F)$.

(2)



A) $sacdfgT$ AND $sbacdfgT$ ARE FLOW-AUGMENTING PATHS.

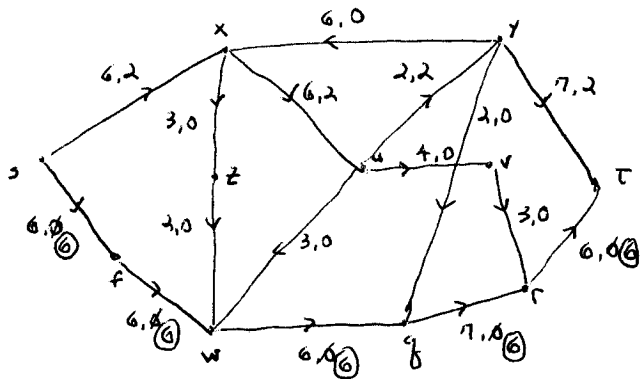
B) USING $sacdfgT$: $\Delta = 1$

LET $C = \{s, T\}$ WHERE $S = \{s, a, b, c\}$

AND $T = \{d, e, f, g, h, T\}$

$cap(C) = C_{sh} + C_{se} + C_{cd} = 4 + 7 + 9 = 20 = val(F)$

(3)



A) $sfwqrT$, $sxuvrT$, $sxzwqrT$ ARE FLOW-AUGMENTING PATHS

B) USING $sfwqrT$: $\Delta = 6$

LET $C = \{s, T\}$ WHERE

$S = \{s, x, f, z, w, u, v, r, q\}$ AND

$T = \{y, T\}$

$cap(C) = C_{uy} + C_{rT} = 2 + 6 = 8 = val(F)$

REMARK TO FIND C, LET S BE THE SET OF VERTICES WHICH CAN BE REACHED FROM s BY FLOW-AUGMENTING PATHS (INCLUDING s ITSELF), AND LET T BE THE REMAINING VERTICES.