(2) LET $S = \{1, 2, \ldots, 10000\}$, A_1 BE THE MULTIPLES OF 4 IN S, A_2 THE MULTIPLES OF 6 IN S, A_3 THE MULTIPLES OF 7 IN S, AND A_4 THE MULTIPLES OF 10 IN S.

$$|\bar{A}_1 \cap \cdots \cap \bar{A}_4| = |S| - \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| - \sum_{i<j<k} |A_i \cap A_j \cap A_k| + |A_1 \cap \cdots \cap A_4|$$

$$= \underbrace{10{,}000}_{} - \underbrace{2{,}500}_{(1)} - \underbrace{1{,}666}_{(2)} - \underbrace{1{,}428}_{(3)} - \underbrace{1{,}000}_{(4)} + \underbrace{833}_{(1,2)} + \underbrace{357}_{(1,3)} + \underbrace{500}_{(1,4)} + \underbrace{238}_{(2,3)} + \underbrace{571}_{(2,4)} + \underbrace{142}_{(3,4)}$$

$$- \underbrace{119}_{(1,2,3)} - \underbrace{166}_{(1,2,4)} - \underbrace{71}_{(1,3,4)} - \underbrace{47}_{(2,3,4)} + \underbrace{23}_{(1,2,3,4)} = \boxed{5{,}667}$$

(3) LET $S = \{1, 2, \ldots, 10000\}$, A THE PERFECT SQUARES IN S, AND B BE THE PERFECT CUBES IN S.

$$|\bar{A} \cap \bar{B}| = |S| - |A| - |B| + |A \cap B| = 10{,}000 - 100 - 21 + 4 = \boxed{9{,}883}$$

(USING THAT $\sqrt{10{,}000} = 100$, $\sqrt[3]{10{,}000} \approx 21.5$, $\sqrt[6]{10{,}000} \approx 4.6$)

(5) LET T BE THE 10-COMBINATIONS OF $\{10 \cdot a, 10 \cdot b, 10 \cdot c, 10 \cdot d\}$, AND

LET A_1 BE THE ELEMENTS OF T WITH AT LEAST 5 b'S,

LET A_2 BE THE ELEMENTS OF T WITH AT LEAST 6 c'S, AND

LET A_3 BE THE ELEMENTS OF T WITH AT LEAST 8 d'S.

THEN $|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| = |T| - \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| - |A_1 \cap A_2 \cap A_3|$

$$= \boxed{\underbrace{\binom{13}{3}}_{|T|} - \underbrace{\binom{8}{3}}_{|A_1|} - \underbrace{\binom{7}{3}}_{|A_2|} - \underbrace{\binom{5}{3}}_{|A_3|}} = \boxed{185}$$

(SINCE ALL DOUBLE INTERSECTIONS ARE EMPTY)

(6) LET T BE THE SET OF OPTIONS IF THERE WERE 12 OF EACH TYPE AVAILABLE, AND LET A_1 BE THE OPTIONS WITH AT LEAST 7 CHOCOLATE, A_2 THE OPTIONS WITH AT LEAST 7 CINNAMON, AND A_3 THE OPTIONS WITH AT LEAST 4 PLAIN.

THEN $|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| = |T| - \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| - |A_1 \cap A_2 \cap A_3|$

$$= \boxed{\underbrace{\binom{14}{2}}_{|T|} - \underbrace{\binom{7}{2}}_{|A_1|} - \underbrace{\binom{7}{2}}_{|A_2|} - \underbrace{\binom{10}{2}}_{|A_3|} + \underbrace{\binom{3}{2}}_{|A_1 \cap A_3|} + \underbrace{\binom{3}{2}}_{|A_2 \cap A_3|}} = \boxed{10}$$

OR CONSIDER CASES, BASED ON HOW MANY PLAIN ARE CHOSEN:

a) 0 PLAIN: 1 OPTION (6 CH., 6 CI.)

b) 1 PLAIN: 2 OPTIONS (6 CH., 5 CI. OR 5 CH., 6 CI.)

c) 2 PLAIN: 3 OPTIONS (6 CH., 4 CI. OR 5 CH., 5 CI. OR 4 CH., 6 CI.)

d) 3 PLAIN: 4 OPTIONS (6 CH., 3 CI. OR 5 CH., 4 CI. OR 4 CH., 5 CI. OR 3 CH., 6 CI.)

TOTAL: $1 + 2 + 3 + 4 = \boxed{10}$ OPTIONS

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(9) $x_1 + x_2 + x_3 + x_4 = 20, \quad 1 \le x_1 \le 6, \quad 0 \le x_2 \le 7, \quad 4 \le x_3 \le 8, \quad 2 \le x_4 \le 6$

Let $y_1 = x_1 - 1$, $y_2 = x_2$, $y_3 = x_3 - 4$, $y_4 = x_4 - 2$ to get

$\underline{y_1 + y_2 + y_3 + y_4 = 13}$ with $0 \le y_1 \le 5$, $0 \le y_2 \le 7$, $0 \le y_3 \le 4$, $0 \le y_4 \le 4$

Let S be the set of nonnegative integral solutions, A_1 the elements of S with $y_1 \ge 6$, A_2 the elements of S with $y_2 \ge 8$, A_3 the elements of S with $y_3 \ge 5$, and A_4 the elements of S with $y_4 \ge 5$.

Then $|\overline{A_1} \cap \cdots \cap \overline{A_4}| = |S| - \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| - \sum_{i<j<k} |A_i \cap A_j \cap A_k| + |A_1 \cap \cdots \cap A_4|$

$$= \boxed{\binom{16}{3} - \underset{1}{\binom{10}{3}} - \underset{2}{\binom{8}{3}} - \underset{3}{\binom{11}{3}} - \underset{4}{\binom{11}{3}} + \underset{1,3}{\binom{5}{3}} + \underset{1,4}{\binom{5}{3}} + \underset{2,3}{\binom{3}{3}} + \underset{2,4}{\binom{3}{3}} + \underset{3,4}{\binom{6}{3}}} = \boxed{96}$$

(17) Let T be the set of permutations of $S = \{3 \cdot a, 4 \cdot b, 2 \cdot c\}$, and let A be the permutations with the a's consecutive, B be the permutations with the b's consecutive, and C be the permutations with the c's consecutive.

Then $|\overline{A} \cap \overline{B} \cap \overline{C}| = |T| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C| - |A \cap B \cap C|$

$$= \boxed{\frac{9!}{3!\,4!\,2!} - \frac{7!}{4!\,2!} - \frac{6!}{3!\,2!} - \frac{8!}{3!\,4!} + \frac{4!}{2!} + \frac{6!}{4!} + \frac{5!}{3!} - 3!} = \boxed{871}$$

(29) Let S be the set of selections of stops for the 10 people without restrictions, and let A_i be the selections of stops where no one exits at stop i, $1 \le i \le 6$.

Then $|\overline{A_1} \cap \cdots \cap \overline{A_6}| = |S| - \sum_i |A_i| + \sum_{i<j} |A_i \cap A_j| - \sum_{i<j<k} |A_i \cap A_j \cap A_k| + \cdots + |A_1 \cap \cdots \cap A_6|$

$$= \boxed{6^{10} - \binom{6}{1} 5^{10} + \binom{6}{2} 4^{10} - \binom{6}{3} 3^{10} + \binom{6}{4} 2^{10} - \binom{6}{5} 1^{10}} = \boxed{16{,}435{,}440}$$