(9) a) IF THE FIRST SQUARE IS WHITE OR BLUE,
THERE ARE 2 CHOICES FOR THE 1ST SQUARE AND h_{n-1} WAYS TO FINISH THE COLORING;

$\underset{1}{\underline{W,B}}\ \underset{2}{\underline{\ \ }}\ \underset{3}{\underline{\ \ }}\ \cdots\ \underset{n}{\underline{\ \ }}$ SO THERE ARE $2h_{n-1}$ POSSIBILITIES.

b) IF THE FIRST SQUARE IS RED, THERE ARE 2 CHOICES FOR THE 2ND SQUARE AND h_{n-2} WAYS WAYS TO FINISH THE COLORING;

$\underset{1}{\underline{R}}\ \underset{2}{\underline{W,B}}\ \underset{3}{\underline{\ \ }}\ \cdots\ \underset{n}{\underline{\ \ }}$ SO THERE ARE $2h_{n-2}$ POSSIBILITIES.

THEN $h_n = 2h_{n-1} + 2h_{n-2}$ WITH $h_0 = 1$, $h_1 = 3$, $h_2 = 8$

$r^2 = 2r + 2$ GIVES $r^2 - 2r - 2 = 0$, SO $r^2 - 2r + 1 = 2 + 1$, $(r-1)^2 = 3$, $r = 1 \pm \sqrt{3}$

THEN $h_n = d(1+\sqrt{3})^n + e(1-\sqrt{3})^n$, AND $h_0 = 1$ GIVES $d + e = 1$ AND
$h_1 = 3$ GIVES $d(1+\sqrt{3}) + e(1-\sqrt{3}) = 3$,
SO $(d-e)\sqrt{3} = 2$ AND $d - e = \frac{2}{\sqrt{3}}$.

SOLVING GIVES $d = \frac{1}{2}\left(1 + \frac{2}{\sqrt{3}}\right) = \frac{1}{2}\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$

AND $e = \frac{1}{2}\left(1 - \frac{2}{\sqrt{3}}\right) = \frac{1}{2}\left(\frac{\sqrt{3}-2}{\sqrt{3}}\right)$,

SO $\boxed{h_n = \frac{2+\sqrt{3}}{2\sqrt{3}}(1+\sqrt{3})^n + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^n}$

(17) $g(x) = (1 + x^2 + x^4 + \cdots)(1 + x + x^2)(1 + x^3 + x^6 + x^9 + \cdots)(1+x)$

$= \frac{1}{1-x^2} \cdot \frac{\cancel{1-x^3}}{1-x} \cdot \frac{1}{\cancel{1-x^3}} \cdot (1+x) = \frac{\cancel{1+x}}{\cancel{(1+x)}(1-x)(1-x)} = \boxed{\frac{1}{(1-x)^2}} = \sum_{n=0}^{\infty} \binom{n+1}{1} x^n$,

SO $h_n = \binom{n+1}{1} = \boxed{n+1}$.

(18) $g(x) = (1 + x^2 + x^4 + \cdots)(1 + x^5 + x^{10} + \cdots)(1 + x + x^2 + \cdots)(1 + x^7 + x^{14} + \cdots)$

$= \boxed{\frac{1}{1-x^2} \cdot \frac{1}{1-x^5} \cdot \frac{1}{1-x} \cdot \frac{1}{1-x^7}}$

(35) $h_n = 3h_{n-2} - 2h_{n-3}$, $h_0 = 1$, $h_1 = 0$, $h_2 = 0$

$r^n = 3r^{n-2} - 2r^{n-3}$ GIVES $r^3 = 3r - 2$, SO $r^3 - 3r + 2 = 0$

THEN $(r-1)(r^2 + r - 2) = 0$, SO $(r-1)^2(r+2) = 0$ AND $r = 1$ OR $r = -2$

THEN $h_n = d(1^n) + e(n \cdot 1^n) + f(-2)^n = d + en + f(-2)^n$.

$h_0 = 1$, SO $d + f = 1$
$h_1 = 0$, SO $d + e - 2f = 0 \longrightarrow 2d + 2e - 4f = 0$
$h_2 = 0$, SO $d + 2e + 4f = 0$ $\quad - \quad d + 2e + 4f = 0$
$\qquad d - 8f = 0$ SO $d = 8f$, $9f = 1$,

$f = \frac{1}{9}$, $d = \frac{8}{9}$, $e = 2f - d$ SO $e = -\frac{2}{3}$;

$\boxed{h_n = \frac{8}{9} - \frac{2}{3}n + \frac{1}{9}(-2)^n}$

(37) LET b_n BE THE NUMBER OF STRINGS OF LENGTH n THAT START WITH 0, c_n BE THE NUMBER OF STRINGS OF LENGTH n THAT START WITH 1, AND d_n BE THE NUMBER OF STRINGS OF LENGTH n THAT START WITH 2.

THEN $\underline{a_n = b_n + c_n + d_n}$ WHERE $\underline{b_n = c_{n-1} + d_{n-1}}$, $\underline{c_n = b_{n-1} + d_{n-1}}$ AND $\underline{d_n = a_{n-1}}$

$\underset{1}{\underline{0}}\ \underset{2}{\underline{1,2}}\ \underset{3}{\underline{\ }}\ \cdots\ \underset{n}{\underline{\ }}$

$\underset{1}{\underline{1}}\ \underset{2}{\underline{0,2}}\ \underset{3}{\underline{\ }}\ \cdots\ \underset{n}{\underline{\ }}$

$\underset{1}{\underline{2}}\ \underset{2}{\underline{\ }}\ \underset{3}{\underline{\ }}\ \cdots\ \underset{n}{\underline{\ }}$

THEREFORE $a_n = (c_{n-1} + d_{n-1}) + (b_{n-1} + d_{n-1}) + a_{n-1}$
$= a_{n-1} + (b_{n-1} + c_{n-1} + d_{n-1}) + d_{n-1} = a_{n-1} + a_{n-1} + d_{n-1}$,

SO $\boxed{a_n = 2a_{n-1} + a_{n-2}}$ WITH $a_0 = 1$, $a_1 = 3$, $a_2 = 7$

$r^2 = 2r + 1$ GIVES $\underline{r^2 - 2r - 1 = 0}$, SO $r^2 - 2r + \underline{1} = 1 + \underline{1}$, $(r-1)^2 = 2$, $\underline{r = 1 \pm \sqrt{2}}$

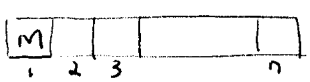
THEN $\underline{a_n = d(1+\sqrt{2})^n + e(1-\sqrt{2})^n}$,

AND $a_0 = 1$ GIVES $\underline{d + e = 1}$ AND $a_1 = 3$ GIVES $d(1+\sqrt{2}) + e(1-\sqrt{2}) = 3$
SO $(d-e)\sqrt{2} = 2$, $\underline{d - e = \sqrt{2}}$

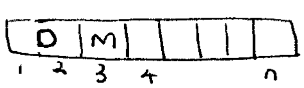
THEN $\underline{d = \frac{1}{2}(1+\sqrt{2})}$ AND $\underline{e = \frac{1}{2}(1-\sqrt{2})}$,

SO $\boxed{a_n = \frac{1}{2}(1+\sqrt{2})^{n+1} + \frac{1}{2}(1-\sqrt{2})^{n+1}}$

(39)
a) IF A MONOMINO COVERS THE FIRST SQUARE, THERE ARE $\underline{h_{n-1}}$ WAYS TO FINISH,



b) IF A DOMINO COVERS THE FIRST 2 SQUARES, THEN A MONOMINO MUST COME NEXT, AND THERE ARE $\underline{h_{n-3}}$ WAYS TO FINISH,



THEREFORE $\boxed{h_n = h_{n-1} + h_{n-3}}$ WITH $\underline{h_0 = 1,\ h_1 = 1,\ h_2 = 2}$.