(3) Choose $n+1$ integers from $\{1, \dots, 2n\}$; then there are 2 of the chosen integers such that one of them divides the other.

PF Write each of the integers in the form $2^k a$ where $k \geq 0$ and a is odd, and then map each integer to the set among $\{1\}, \{3\}, \{5\}, \dots, \{2n-1\}$ which contains its odd part a.
Since there are $n+1$ integers and only n sets, two of the integers must map to the same set $\{a\}$ by the PH principle, therefore they can be written in the form $2^k a$ and $2^l a$ where we can assume that $k < l$, so $2^k a$ divides $2^l a$.

(4) Map each of the integers to the set among $\{1,2\}, \{3,4\}, \dots, \{2n-1, 2n\}$ to which it belongs. Since there are $n+1$ integers and only n sets, the PH principle implies that at least one of these sets contains 2 of the integers.

(9) There are $2^{10} - 1 = 1023$ nonempty subsets of the group of the 10 people, and the sum of the ages of the people in any subset is in $[600] = \{1, \dots, 600\}$.
If we map each subset to its age sum, by the PH principle there must be two subsets with the same age sum (since $1023 > 600$); and then we can remove the people who are in both subsets to get 2 disjoint groups with the same age sum.

(14) Worst-case scenario: we first pick 11 of each of the 4 types of the fruit. If we then pick 1 more piece of fruit, we are assured of having at least a dozen of the same kind, so after 45 minutes we will have what we want.

OR Let $q_i = 12$ for $1 \leq i \leq 4$. By the strong form of the PH principle, if we have $(q_1 + \dots + q_4) - 4 + 1 = 48 - 4 + 1 = 45$ pieces of fruit, then we must have at least 12 of the same type.

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(15) Map each number in $\{a_1, \dots, a_{n+1}\}$ to the number in $\{0, 1, 2, \dots, n-1\}$ which gives its remainder when divided by n. Since there are $n+1$ numbers and only n remainders, by the PH principle there are two integers a_i and a_j which have the same remainder when divided by n. Therefore $a_i = kn + r$ and $a_j = ln + r$ (for some integers k, l, r), so $a_i - a_j$ is divisible by n.

(16) Map each person to the number of acquaintances they have.
There are two possibilities to consider:

1) If every person has an acquaintance, the number of acquaintances for each person is in $\{1, \dots, n-1\}$. Then the number of acquaintances for each person... Since there are n people and only $n-1$ possibilities for the number of acquaintances, the PH principle implies that there are 2 people with the same number of acquaintances.

2) If there is a person with no acquaintances, the number of acquaintances for each person is in $\{0, 1, \dots, n-2\}$. Since there are n people and only $n-1$ possibilities for the number of acquaintances, by the PH principle, there must be two people with the same number of acquaintances.

MATH 145

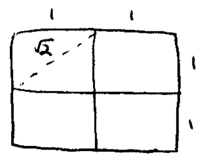
CH. 3 SOLUTIONS

(10) LET a_n BE THE NUMBER OF HOURS THE CHILD WATCHES TV DURING THE FIRST n DAYS.
THEN $1 \le a_1 < a_2 < a_3 < \cdots < a_{49} \le 77$ BY THE GIVEN CONDITIONS, AND

$$21 \le a_1 + 20 < a_2 + 20 < a_3 + 20 < \cdots < a_{49} + 20 \le 97.$$

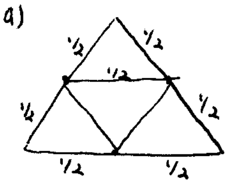
SINCE THE 98 NUMBERS $\{a_1, \ldots, a_{49}, a_1+20, \ldots, a_{49}+20\}$ ARE IN $[97] = \{1, \ldots, 97\}$, TWO OF THE NUMBERS MUST BE EQUAL BY THE PH PRINCIPLE.
SINCE $a_i \ne a_j$ FOR $i \ne j$ AND $a_i + 20 \ne a_j + 20$ FOR $i \ne j$, $a_i = a_j + 20$ FOR SOME $i > j$.
THEREFORE THE CHILD WATCHES TV FOR EXACTLY 20 HOURS ON DAYS $j+1, j+2, \ldots, i$.

(18)



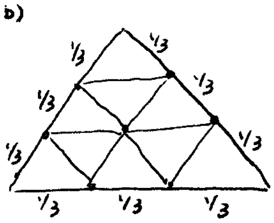
CUT THE SQUARE INTO 4 1x1 SQUARES; SINCE 5 POINTS ARE DISTRIBUTED INTO THE 4 BOXES, BY THE PH PRINCIPLE THERE MUST BE 2 POINTS IN THE SAME SQUARE. THEREFORE THE DISTANCE BETWEEN THEM IS AT MOST $\sqrt{2}$.

(19) a)



CUT THE TRIANGLE INTO 4 EQUILATERAL TRIANGLES WITH SIDE LENGTH $1/2$. SINCE THERE ARE 5 POINTS AND 4 TRIANGLES, THERE MUST BE 2 POINTS IN THE SAME TRIANGLE BY THE PH PRINCIPLE; SO THE DISTANCE BETWEEN THEM IS AT MOST $1/2$.

b)



CUT THE TRIANGLE INTO 9 EQUILATERAL TRIANGLES WITH SIDE LENGTH $1/3$. SINCE THERE ARE 10 POINTS AND 9 TRIANGLES, THERE MUST BE 2 POINTS IN THE SAME TRIANGLE BY THE PH PRINCIPLE; SO THE DISTANCE BETWEEN THEM IS AT MOST $1/3$.