MATH 145 — TEST 2 REVIEW PROBLEMS

(1) Let G be a graph with n vertices.

A) If G is connected, what is its minimum number of edges?

B) If G is not connected, what is its maximum number of edges?

(2) Find the following, and justify your answers:

A) A graph that has a Hamilton cycle but does not have a closed Eulerian trail.

B) A graph that has a closed Eulerian trail but does not have a Hamilton cycle.

(3) A) Show that there is no graph with degree sequence $(6,5,4,4,3,1,1)$.

B) Draw a general graph with degree sequence $(6,5,4,4,3,1,1)$.

(4) Prove that a graph $G=(V,E)$ is not connected iff there is a subset U of V (with $U \neq \emptyset$, $U \neq V$) such that $xy \notin E$ whenever $x \in U$ and $y \notin U$.

(5) For $n \geq 3$, let G_n be the graph obtained from K_n by deleting an edge. Find the values of n for which G_n has an Eulerian trail, and justify your answer.

(6) In the following, use the definition that a tree is a connected graph which has no cycles:

A) Prove that a graph G is a tree iff G is connected, and every edge of G is a bridge.

B) Prove that a graph G is a tree iff every pair of distinct vertices is connected by a unique path.

(7) Draw a tree with degree sequence $(5,3,3,3,2,1,1,1,1)$, or explain why this is impossible.

(8) Suppose there are 13 people in a room. Show that either there is at least one person who knows at least 6 others, or there is a group of 3 people, none of whom know each other.

(9) Assume that there are 10 users of Facebooook. If user k has k friends for $1 \leq k \leq 9$, who are the friends of user 10?

(10) Let G be a graph with $2m$ vertices that doesn't contain a triangle C_3 as a subgraph. Use induction on m to prove that G has at most m^2 edges.

(11) If C_n is the cycle graph of order n, show that its chromatic polynomial is $(k-1)^n + (-1)^n (k-1)$.