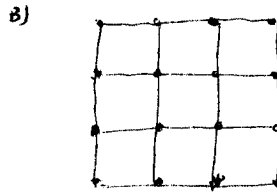
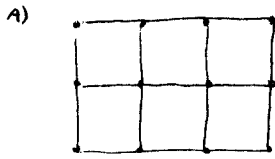


① Let  $G$  be a graph with  $n$  vertices and  $e$  edges.

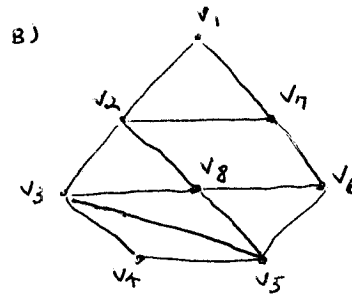
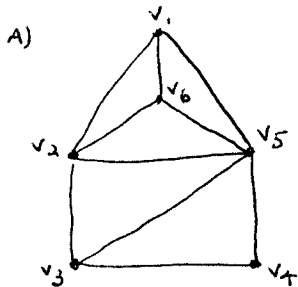
A) Use induction on  $e$  to show that  $G$  has at least  $n - e$  connected components.

B) Use part A) to show that if  $G$  is connected, then it has at least  $n - 1$  edges.

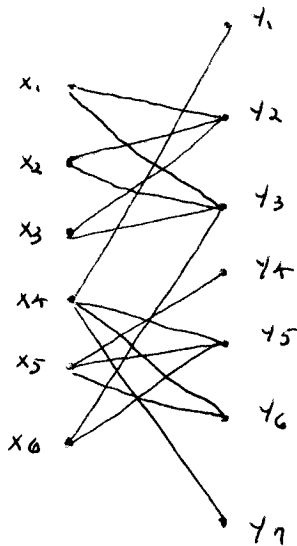
② Determine whether the following graphs have Hamilton cycles:



③ Apply the greedy vertex-coloring algorithm to color the following graphs:  
 (use  $v_i \rightarrow c_j$  to indicate that vertex  $v_i$  is colored with color  $c_j$ )



④ a) Show that there is no matching of  $X$  into  $Y$  in the bipartite graph below.  
 b) Find a maximum matching in the graph.



⑤ If you deal a standard deck of 52 cards into 13 piles of 4 cards each, show that it is always possible to select 1 card from each pile so that you get one card of each rank (one ace, one king, one jack, etc.).