

USE GENERATING FUNCTIONS OR EXPONENTIAL GENERATING FUNCTIONS TO SOLVE THE FOLLOWING PROBLEMS:

- ① FIND THE NUMBER OF WAYS TO SELECT 10 BALLS FROM A COLLECTION OF 6 RED, 10 ORANGE, AND 10 GREEN BALLS IF AN EVEN NUMBER OF GREEN BALLS MUST BE SELECTED.
- ② FIND THE NUMBER OF WAYS TO PLACE 20 QUARTERS IN 6 BOXES, SO THAT THE FIRST BOX CONTAINS BETWEEN 1 AND 5 QUARTERS AND EACH OF THE OTHER BOXES CONTAIN AT LEAST 2 QUARTERS.
- ③ FIND THE NUMBER OF WAYS TO SELECT 50 MARBLES FROM A BAG CONTAINING 50 RED, 43 BLUE, 29 WHITE, AND 24 GREEN MARBLES IF AN EVEN NUMBER OF RED MARBLES AND AN ODD NUMBER OF BLUE MARBLES MUST BE SELECTED.
- ④ IF a_n IS THE NUMBER OF INTEGRAL SOLUTIONS OF $2x + y + z = n$ WITH $x, y \geq 0$ AND $z \geq 1$, FIND A FORMULA FOR a_n .
- ⑤ FIND THE NUMBER OF SEQUENCES OF n DIGITS, EACH OF WHICH IS EITHER 1, 2, 3, OR 4, SUCH THAT 1, 2, AND 3 EACH OCCUR AN ODD NUMBER OF TIMES.
- ⑥ DETERMINE THE NUMBER OF n -DIGIT SEQUENCES FOR WHICH EACH DIGIT IS IN $\{1, \dots, 5\}$ AND 1 AND 2 EACH OCCUR AN EVEN NUMBER OF TIMES AND 5 OCCURS AT LEAST ONCE.