3.3 - Solutions

5. \( f(x) = \frac{2x^4}{x^2 + 1} \)  
   \( f'(x) = -2x \left( \frac{x^2 + 1}{x^4} \right) \)  
   \( f''(x) = \frac{4x^2 - 2}{(x^2 + 1)^3} \)  
   \( f''(x) = 4x \left( \frac{2x^2 - 4}{(x^2 + 1)^3} \right) \)  
   \( f''(x) = \frac{4x(2x - 4)}{(x^2 + 1)^3} \)  
   \( f''(0) = -\frac{16}{12^3} \)

Graph of \( f(x) \) on \((-\infty, 2) \) and \((2, \infty) \)

6. \( f(x) = \frac{1}{x^2 + 1} \)  
   \( f'(x) = -\frac{2x}{(x^2 + 1)^2} \)  
   \( f''(x) = \frac{4x}{(x^2 + 1)^3} \)  
   \( f''(x) = \frac{4x(2x - 4)}{(x^2 + 1)^3} \)  
   \( f''(0) = -\frac{16}{12^3} \)

7. \( f(x) = (x-1)^3 (x-5) \)  
   \( f'(x) = 3(x-1)^2 (x-5) + 3(x-1)(x-5)^2 \)  
   \( f''(x) = 12(x-1)^2 (x-5) + 12(x-1)(x-5)^2 \)  
   Points of Inflection: \((1, 0)\) and \((3, -16)\)

8. \( g(x) = (x-2)(x+1)^2 \)  
   \( g'(x) = (x-2)(2x+2) + 1(x+1)^2 \)  
   \( g''(x) = 3x - 3 \)  
   \( g''(x) = 6x \)  
   \( g''(0) = 0 \)  
   \( g''(1) = 3 \)

9. \( f(x) = \frac{1}{1 + x^2} \)  
   \( f'(x) = -\frac{2x}{(1 + x^2)^2} \)  
   \( f''(x) = \frac{4x}{(1 + x^2)^3} \)  
   Points of Inflection: \((-\frac{\sqrt{2}}{2}, 3)\) and \((\frac{\sqrt{2}}{2}, 3)\)
48. (Other answers are possible)

50.

a) \( f'(x) > 0 \) on \((0, 2)\), where \( f \) is increasing.

b) \( f'(x) < 0 \) on \((-\infty, 0)\) and \((2, \infty)\), where \( f \) is decreasing.

c) \( f' \) is increasing on \((-\infty, 1)\), where the graph of \( f \) is concave up.

d) \( f' \) is decreasing on \((1, \infty)\), where the graph of \( f \) is concave down.

65. \( N = -T^3 + 12T^2 \), \( 0 \leq T \leq 12 \)

a) \( N'(T) = -3T^2 + 24T = 3T(T - 8) = 0 \) \( \Rightarrow T = 0 \) or \( T = 8 \)

\( N(8) = 256 \) \( \Rightarrow \) the max. number projected to be infected is 25,600 (since \( N \) is in hundreds)

\( N(0) = 0 \)

\( N(12) = 0 \)

b) The virus is spreading most rapidly when the rate of change \( N'(T) \) is a maximum:

\( N''(T) = -6T + 24 = 0 \) \( \Rightarrow T = 4 \)

\( N'(4) = 48 \) \( \Rightarrow \) the virus is spreading most rapidly when \( T = 4 \) weeks

\( N'(0) = 0 \)

\( N'(12) = -144 \)