f(x) = x^3/4

\[ f'(x) = \lim_{t \to x} \frac{f(t)-f(x)}{t-x} = \lim_{t \to x} \frac{t^{3/4} - x^{3/4}}{t-x} = \lim_{t \to x} \frac{(\sqrt[4]{t})^3 - (\sqrt[4]{x})^3}{(\sqrt[4]{t})^3 - (\sqrt[4]{x})^3} \]

\[ = \lim_{t \to x} \frac{(\sqrt[4]{t} - \sqrt[4]{x})(t + \sqrt[4]{tx} + x)}{(\sqrt[4]{t} - \sqrt[4]{x})(\sqrt[4]{t} + \sqrt[4]{x})} = \lim_{t \to x} \frac{t + \sqrt[4]{tx} + x}{\sqrt[4]{t} + \sqrt[4]{x}} = \frac{3x}{\sqrt[4]{x}} = 3 \sqrt[4]{x} \]

f(x) = \frac{10x}{x^2 + 4} \at\ (1,2)

m = f'(1) = \lim_{t \to 1} \frac{f(t) - f(1)}{t-1} = \lim_{t \to 1} \frac{10t}{t^2 + 4} - \frac{2}{t-1} = \lim_{t \to 1} \frac{10t - 2(t^2 + 4)}{(t-1)(t^2 + 4)} \]

\[ = \lim_{t \to 1} \frac{-2t^2 + 10t - 8}{(t-1)(t^2 + 4)} = \lim_{t \to 1} \frac{(t-1)(-2t + 8)}{(t-1)(t^2 + 4)} = \lim_{t \to 1} \frac{-2t + 8}{t^2 + 4} = \frac{6}{5} \]

f(x) = x^2 + \frac{8}{x} \at\ (1,9)

m = f'(1) = \lim_{t \to 1} \frac{f(t) - f(1)}{t-1} = \lim_{t \to 1} \frac{t^2 + \frac{8}{t} - 9}{t-1} = \lim_{t \to 1} \frac{T^2 + \frac{8}{t} - 9}{(t-1)(t)} \]

\[ = \lim_{t \to 1} \frac{T^2 - 9t + 8}{T(t-1)} = \lim_{t \to 1} \frac{(T-1)(T^2 + T - 8)}{T(T-1)} = \lim_{t \to 1} \frac{T^2 + T - 8}{T} = \frac{-6}{T} = -6 \]

f(x) = |x|

f'(0) = \lim_{t \to 0} \frac{f(t) - f(0)}{t-0} = \lim_{t \to 0} \frac{|t|}{t} = \lim_{t \to 0} \frac{1}{} \text{ does not exist} \]

Since
\[ \lim_{t \to 0^+} \frac{|t|}{t} = \lim_{t \to 0^+} \frac{t}{t} = \lim_{t \to 0^+} 1 = 1 \text{ and } \]
\[ \lim_{t \to 0^-} \frac{|t|}{t} = \lim_{t \to 0^-} \frac{-t}{t} = \lim_{t \to 0^-} (-1) = -1. \]

f(x) = \frac{3}{\sqrt{x}}

f'(0) = \lim_{t \to 0} \frac{f(t) - f(0)}{t-0} = \lim_{t \to 0} \frac{\frac{3}{\sqrt{t}}}{t} = \lim_{t \to 0} \frac{3}{\sqrt{t} \cdot t} = \lim_{t \to 0} \frac{1}{t^{3/2}} \text{ does not exist} \]

Since it has the form: \( \frac{1}{0} \).

(Since it has the signs: \( \frac{+}{+} \) because \( t^{3/2} = (\sqrt[4]{t})^3 > 0 \) for \( t \neq 0 \),

\[ \lim_{t \to 0} \frac{1}{t^{3/2}} = \infty \] and this indicates that the graph has a vertical tangent line at \( (0,0) \).)
48. \( f(x) = x^2 - x \); find an equation of the tangent line which is parallel to \( x + 2y - 6 = 0 \).

Since \( x + 2y - 6 = 0 \), or \( y = -\frac{1}{2}x + 3 \), has slope \( m = -\frac{1}{2} \),

the tangent line is parallel to this line when its slope \( f'(x) \) is also \( -\frac{1}{2} \):

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left[ \frac{(x+h)^2 - x^2}{h} \right] = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \left( 2x + h \right) = 2x.
\]

Then \( f'(x) = -\frac{1}{2} \) gives \( 2x = -\frac{1}{2} \), \( x = \frac{1}{4} \), \( x = -\frac{1}{2} \) and

\( y = f(x) = \left( \frac{1}{2} \right)^2 - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} \) so \( y = -\frac{3}{16} \).

Therefore, the tangent line has equation \( y + \frac{3}{16} = -\frac{1}{4} \left( x - \frac{1}{2} \right) \) or \( y = -\frac{1}{2}x - \frac{1}{16} \).

50. \( y = \sqrt{x^2 - 9} \)

\( f(x) = \sqrt{x^2 - 9} \) is differentiable at all real numbers except \( -3 \) and \( 3 \),

since the graph of \( f \) has a sharp edge when \( x = -3 \) and \( x = 3 \).

51. \( y = (x - 3)^{3/2} \)

\( f(x) = (x - 3)^{3/2} \) is differentiable at all real numbers except \( 3 \),

since the graph of \( f \) has a sharp edge and a vertical tangent line when \( x = 3 \).

53. \( y = \begin{cases} x^3 + 3, & \text{if } x < 0 \\ x^3 - 3, & \text{if } x \geq 0 \end{cases} \)

\( f(x) = \begin{cases} x^3 + 3, & \text{if } x < 0 \\ x^3 - 3, & \text{if } x \geq 0 \end{cases} \)

is differentiable at all real numbers except \( 0 \),

since the graph of \( f \) has a break when \( x = 0 \) (so \( 0 \) is a jump discontinuity for \( f \)).