Test 3 Review Problems

1. If \( f(x) = \frac{x^2}{x-5} \), find the relative extrema for \( f \).

2. Let \( f(x) = 3x^5 - 10x^3 \), so \( f'(x) = 15x^4(x^2-2) \) and \( f''(x) = 60x(x^2-1) \).
   a) Find the open intervals on which \( f \) is increasing or decreasing.
   b) Find the open intervals on which the graph of \( f \) is concave up or concave down.

3. If \( f(x) = 3x^{1/3} - 2x^{1/2} + 2 \), find the critical numbers and relative extrema for \( f \).

4. Find the absolute extrema for \( f(x) = \sin x + \sqrt{3} \cos x \) on \( \left[ 0, \frac{\pi}{2} \right] \).

5. A rancher has 360 ft of fencing to enclose 4 adjacent rectangular corrals. Find the values of \( x \) and \( y \) which will maximize the total enclosed area.

6. Sketch the graph of a rational function \( f \) with the following properties:
   a) \( x = 1 \) and \( x = 3 \) are vertical asymptotes.
   b) \( y = x \) is a slanted asymptote.
   c) \( f(0) = 3 \) is a rel. max.
   d) \( f(3) = 2 \) is a rel. min.
   e) \( (3, -1) \) is a point of inflection.

7. Let \( f(x) = \frac{x^4+4}{(x+2)^2} \), so \( f'(x) = \frac{4(x+2)}{(x+2)^3} \) and \( f''(x) = \frac{8(4-x)}{(x+2)^4} \).
   a) Find equations for the asymptotes to the graph of \( f \).
      Vertical: ________  Horizontal: ________
   b) Find the open intervals on which \( f \) is increasing or decreasing.
   c) Find the open intervals on which the graph of \( f \) is concave up or concave down.
   d) Sketch the graph of \( f \), showing all asymptotes, relative extrema, points of inflection, and intercepts.

8. A cylindrical container is to be made from 150 ft \(^2 \) of material. Find the radius and height of the container which will have the largest volume.

9. Just set up a function of 1 variable to be maximized or minimized in the following problems:
   a) Find the area of the largest possible triangle that can be inscribed in a circle of radius 8 in.

10. The combined perimeter of an equilateral triangle and a square is 30 cm. Find the dimensions of the triangle and square which will give a minimum total area.