MATH 16B SOLUTIONS – SEC. 4.6 AND 5.1

4.6 – (31) COMPOUNDING n TIMES A YEAR AT AN ANNUAL INTEREST RATE OF r: $\underline{A = P\left(1+\frac{r}{n}\right)^{nt}}$

COMPOUNDING ANNUALLY AT AN ANNUAL INTEREST RATE OF i: $\underline{A = P(1+i)^t}$

THEN $\cancel{P}\left(1+\frac{r}{n}\right)^{nt} = \cancel{P}(1+i)^t \implies \left(1+\frac{r}{n}\right)^{nt} = (1+i)^t \implies \left(1+\frac{r}{n}\right)^n = 1+i,$

(TAKING BOTH SIDES TO THE $\frac{1}{t}$ POWER, OR JUST USING $t=1$)

SO $\underline{i = \left(1+\frac{r}{n}\right)^n - 1}$

(35) ASSUMING CONTINUOUS COMPOUNDING, $A = Pe^{rt}$; SO THE AMOUNT DOUBLES WHEN

$\cancel{P}e^{rt} = 2\cancel{P}, \quad e^{rt} = 2, \quad rt = \ln 2, \quad t = \frac{\ln 2}{r} = \frac{100 \ln 2}{100r} \approx \frac{69.3}{100r} \approx \underline{\frac{70}{100r}}$

(WHERE $100r$ IS THE ANNUAL INTEREST RATE WRITTEN AS A PERCENT).

FOR EXAMPLE, AN INVESTMENT AT 5% COMPOUNDED CONTINUOUSLY WILL DOUBLE IN ABOUT 14 YEARS, AND AN INVESTMENT AT 7% COMPOUNDED CONTINUOUSLY WILL DOUBLE IN ABOUT 10 YEARS.

5.1 – (51) $f'(x) = 6x(x-1);\quad f(1) = -1$

$\underline{f(x)} = \int 6x(x-1)\,dx = \int (6x^2 - 6x)\,dx = \underline{2x^3 - 3x^2 + C}$, SO

$f(1) = 2 - 3 + C = -1 \implies \underline{C = 0}$ AND SO $\boxed{f(x) = 2x^3 - 3x^2}$

(53) $f'(x) = \dfrac{2-x}{x^3},\ x > 0;\quad f(2) = \frac{3}{4}$

$f(x) = \int \dfrac{2-x}{x^3}\,dx = \int (2-x)x^{-3}\,dx = \int (2x^{-3} - x^{-2})\,dx = \underline{-x^{-2} + x^{-1} + C} = \underline{-\frac{1}{x^2} + \frac{1}{x} + C}$

$f(2) = -\frac{1}{4} + \frac{1}{2} + C = \frac{3}{4} \implies \underline{C = \frac{1}{2}}$, SO $\boxed{f(x) = -\frac{1}{x^2} + \frac{1}{x} + \frac{1}{2}}$

(57) $f'(x) = 6\sqrt{x} - 10;$ PASSES THROUGH $(4, 2)$

$f(x) = \int (6\sqrt{x} - 10)\,dx = \int (6x^{1/2} - 10)\,dx = \underline{4x^{3/2} - 10x + C}$

$f(4) = 4 \cdot 2^3 - 10(4) + C = C - 8 = 2 \implies \underline{C = 10}$, SO $\boxed{f(x) = 4x^{3/2} - 10x + 10}$

(75) 1) $\underline{v(t)} = \int a(t)\,dt = \int -32\,dt = -32t + C = \underline{-32t + 60}$ SINCE $C = v(0) = 60$.

2) $\underline{s(t)} = \int v(t)\,dt = \int (-32t + 60)\,dt = -16t^2 + 60t + D = \underline{-16t^2 + 60t}$

(ASSUMING THAT IT STARTS AT GROUND LEVEL, SO $D = s(0) = 0$).

THE BALL REACHES $\underline{\text{MAX. HEIGHT}}$ WHEN $\underline{v(t) = 0}$,

SO $-32t + 60 = 0$ GIVES $32t = 60$ AND $\underline{t} = \frac{60}{32} = \underline{\frac{15}{8}}$ SEC.

THEN THE MAX. HEIGHT IS

$s\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) = -16 \cdot \frac{225}{64} + \frac{225}{2} = -\frac{225}{4} + \frac{225}{2} = \boxed{\frac{225}{4}\text{ FT}} = \boxed{56.25\text{ FT}}$

MATH 16B

SOLUTIONS – SEC. 5.2 AND 4.6

5.2 – (15) $\int x\,(x^2-1)^7\,dx$ LET $u = x^2 - 1$, $du = 2x\,dx$

$= \frac{1}{2}\int (x^2-1)^7 \cdot 2x\,dx = \frac{1}{2}\int u^7\,du = \frac{1}{2}\left(\frac{u^8}{8}\right) + C = \boxed{\frac{1}{16}(x^2-1)^8 + C}$

(17) $\int \frac{x^2}{(1+x^3)^2}\,dx$ LET $u = 1 + x^3$, $du = 3x^2\,dx$

$= \frac{1}{3}\int \frac{1}{(1+x^3)^2}\cdot 3x^2\,dx = \frac{1}{3}\int \frac{1}{u^2}\,du = \frac{1}{3}\int u^{-2}\,du = \frac{1}{3}\left(-u^{-1}\right) + C = \boxed{-\frac{1}{3}(1+x^3)^{-1} + C}$

(19) $\int \frac{x+1}{(x^2+2x-3)^2}\,dx$ LET $u = x^2+2x-3$, $du = (2x+2)\,dx = 2(x+1)\,dx$

$= \frac{1}{2}\int \frac{1}{(x^2+2x-3)^2}\cdot 2(x+1)\,dx = \frac{1}{2}\int \frac{1}{u^2}\,du = \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}(-u^{-1}) + C = \boxed{-\frac{1}{2}(x^2+2x-3)^{-1} + C}$

(25) $\int \frac{4y}{\sqrt{1+y^2}}\,dy$ LET $u = 1 + y^2$, $du = 2y\,dy$

$= \frac{4}{2}\int \frac{1}{\sqrt{1+y^2}}\cdot 2y\,dy = 2\int \frac{1}{\sqrt{u}}\,du = 2\int u^{-1/2}\,du = 2\left(2u^{1/2}\right) + C = \boxed{4\sqrt{1+y^2} + C}$

(35) $\int x\,(6x^2-1)^3\,dx$ LET $u = 6x^2 - 1$, $du = 12x\,dx$

$= \frac{1}{12}\int (6x^2-1)^3\cdot 12x\,dx = \frac{1}{12}\int u^3\,du = \frac{1}{12}\left(\frac{u^4}{4}\right) + C = \boxed{\frac{1}{48}(6x^2-1)^4 + C}$

(37) $\int x^2\,(2-3x^3)^{3/2}\,dx$ LET $u = 2 - 3x^3$, $du = -9x^2\,dx$

$= \left(-\frac{1}{9}\right)\int (2-3x^3)^{3/2}\,(-9)\,x^2\,dx = -\frac{1}{9}\int u^{3/2}\,du = -\frac{1}{9}\left(\frac{2}{5}u^{5/2}\right) + C$

$= \boxed{-\frac{2}{45}(2-3x^3)^{5/2} + C}$

4.6 – (19) $y = Ce^{kt}$ WHEN $t = 5715$, $y = \frac{1}{2}C$; SO $\cancel{C}e^{5715k} = \frac{1}{2}\cancel{C}$, $e^{5715k} = \frac{1}{2}$, $\ln e^{5715k} = \ln .5$,

$5715k = \ln .5$, $k = \frac{1}{5715}\ln .5$

$y = Ce^{\left(\frac{1}{5715}\ln .5\right)t} = C\left(e^{\ln .5}\right)^{\frac{t}{5715}} = C\left(\frac{1}{2}\right)^{\frac{t}{5715}}$

WHEN $y = .15C$, $\cancel{C}\left(\frac{1}{2}\right)^{\frac{t}{5715}} = .15\cancel{C}$, $\left(\frac{1}{2}\right)^{\frac{t}{5715}} = .15$, $\ln\left(\frac{1}{2}\right)^{\frac{t}{5715}} = \ln .15$,

$\frac{t}{5715}\ln .5 = \ln .15$, $t = \boxed{\frac{5715\ln .15}{\ln .5}\text{ YR}} \approx 15{,}642$ YR