① A) $\displaystyle\int_1^e \frac{30}{x(2\ln x+3)^2}\,dx$ LET $\underline{u=2\ln x+3}$, $du=\frac{2}{x}\,dx$ IF $x=1$, $u=2\cdot 0+3=3$; $x=e$, $u=2\cdot 1+3=5$

$$=\frac{30}{2}\int_1^e \frac{1}{(2\ln x+3)^2}\cdot\frac{2}{x}\,dx = 15\int_3^5 \frac{1}{u^2}\,du = 15\left[-\frac{1}{u}\right]_3^5 = 15\left(-\frac{1}{5}-\left(-\frac{1}{3}\right)\right) = -3+5 = \boxed{2}$$

B) $\displaystyle\int_0^4 \frac{3x}{\sqrt{2x+1}}\,dx$ LET $\underline{u=\sqrt{2x+1}}$, SO $x=\frac{1}{2}(u^2-1)$, $dx=u\,du$ IF $x=0$, $u=1$; $x=4$, $u=3$

$$=\int_1^3 \frac{3\cdot\frac{1}{2}(u^2-1)}{\cancel{u}}\cdot\cancel{u}\,du = \frac{3}{2}\int_1^3 (u^2-1)\,du = \frac{3}{2}\left[\frac{u^3}{3}-u\right]_1^3 = \frac{3}{2}\left((9-3)-\left(\frac{1}{3}-1\right)\right) = \frac{3}{2}\cdot\frac{20}{3} = \boxed{10}$$

OR LET $\underline{u=2x+1}$, $x=\frac{1}{2}(u-1)$, $dx=\frac{1}{2}\,du$ IF $x=0$, $u=1$; $x=4$, $u=9$ TO GET

$$\int_1^9 \frac{3\cdot\frac{1}{2}(u-1)}{\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{3}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\,du = \frac{3}{4}\int_1^9 (u^{1/2}-u^{-1/2})\,du = \frac{3}{4}\left[\frac{2}{3}u^{3/2}-2u^{1/2}\right]_1^9$$

$$=\frac{3}{4}\left(\left(\frac{2}{3}\cdot 27-2\cdot 3\right)-\left(\frac{2}{3}-2\right)\right) = \frac{3}{4}\left(\frac{40}{3}\right) = \boxed{10}$$

C) $\displaystyle\int_1^{e^3} \frac{\ln x}{x^2}\,dx$ LET $\underline{u=\ln x,\ dv=\frac{1}{x^2}\,dx}$; $du=\frac{1}{x}\,dx$, $v=-\frac{1}{x}$

$$=\left[-\frac{\ln x}{x}-\int -\frac{1}{x^2}\,dx\right]_1^{e^3} = \left[-\frac{\ln x}{x}-\frac{1}{x}\right]_1^{e^3} = \left(-\frac{3}{e^3}-\frac{1}{e^3}\right)-(0-1) = \boxed{1-\frac{4}{e^3}}$$

D) $\displaystyle\int_0^{\pi/3} \frac{10\sec\theta\tan\theta}{2\sec\theta-1}\,d\theta$ LET $\underline{u=2\sec\theta-1}$, $du=2\sec\theta\tan\theta\,d\theta$ IF $\theta=0$, $u=2\cdot 1-1=1$; $\theta=\frac{\pi}{3}$, $u=2\cdot 2-1=3$

$$=\frac{10}{2}\int_0^{\pi/3} \frac{1}{2\sec\theta-1}\cdot 2\sec\theta\tan\theta\,d\theta = 5\int_1^3 \frac{1}{u}\,du = 5\Big[\ln u\Big]_1^3 = 5(\ln 3-\overset{0}{\ln 1}) = \boxed{5\ln 3}$$

② $\displaystyle\int x^2\cos 5x\,dx = \boxed{\frac{1}{5}x^2\sin 5x+\frac{2}{25}x\cos 5x-\frac{2}{125}\sin 5x+C} \longleftarrow$

u		dv
x^2	$+$	$\cos 5x\,dx$
$2x$	$-$	$\frac{1}{5}\sin 5x$
2	$+$	$-\frac{1}{25}\cos 5x$
0		$-\frac{1}{125}\sin 5x$

OR LET $\underline{u=x^2,\ dv=\cos 5x\,dx}$ TO GET; $du=2x\,dx$, $v=\frac{1}{5}\sin 5x$

$\frac{1}{5}x^2\sin 5x-\frac{2}{5}\int x\sin 5x\,dx$ LET $\underline{u=x,\ dv=\sin 5x\,dx}$; $du=dx$, $v=-\frac{1}{5}\cos 5x$

$$=\frac{1}{5}x^2\sin 5x-\frac{2}{5}\left[-\frac{1}{5}x\cos 5x-\left(-\frac{1}{5}\right)\int\cos 5x\,dx\right]$$

$$=\boxed{\frac{1}{5}x^2\sin 5x+\frac{2}{25}x\cos 5x-\frac{2}{25}\left(\frac{1}{5}\sin 5x\right)+C} = \boxed{\frac{1}{5}x^2\sin 5x+\frac{2}{25}x\cos 5x-\frac{2}{125}\sin 5x+C}$$

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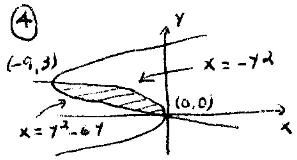
③ $\displaystyle\bar{f} = \frac{1}{4-0}\int_0^4 \frac{22t}{\sqrt{3t^2+1}}\,dt$ LET $\underline{u=3t^2+1}$, $du=6t\,dt$ IF $t=0$, $u=1$; $t=4$, $u=49$

$$=\frac{1}{4}\cdot\frac{22}{6}\int_0^4 \frac{1}{\sqrt{3t^2+1}}\cdot 6t\,dt = \frac{11}{12}\int_1^{49}\frac{1}{\sqrt{u}}\,du = \frac{11}{12}\int_1^{49} u^{-1/2}\,du = \frac{11}{12}\left[2u^{1/2}\right]_1^{49}$$

$$=\frac{11}{6}(\sqrt{49}-\sqrt{1}) = \frac{11}{6}(7-1) = \boxed{11\text{ cm/sec}}$$

(4)



$y^2 - 6y = -y^2, \quad 2y^2 - 6y = 0, \quad 2y(y-3) = 0, \quad \underline{y=0} \text{ or } \underline{y=3}$

$$A = \int_0^3 \left(-y^2 - (y^2 - 6y)\right) dy = \int_0^3 (6y - 2y^2)\, dy = \left[3y^2 - \frac{2}{3}y^3\right]_0^3 = 27 - \frac{2}{3}(27) - 0$$

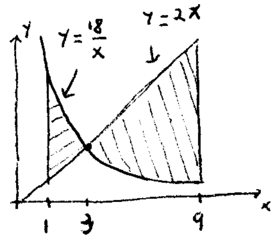
$$= 27 - 18 = \boxed{9}$$

(5) $\displaystyle\int \frac{x^2}{(2x+5)^3}\, dx$ Let $\underline{u = 2x+5}$, $x = \frac{1}{2}(u-5)$, $dx = \frac{1}{2}du$

$$= \int \frac{\frac{1}{4}(u-5)^2}{u^3} \cdot \frac{1}{2} du = \frac{1}{8}\int \frac{u^2 - 10u + 25}{u^3}\, du = \frac{1}{8}\int \left(\frac{u^2}{u^3} - \frac{10u}{u^3} + \frac{25}{u^3}\right) du = \frac{1}{8}\int \left(\frac{1}{u} - 10u^{-2} + 25u^{-3}\right) du$$

$$= \frac{1}{8}\left[\ln|u| + 10u^{-1} - \frac{25}{2}u^{-2}\right] + C = \boxed{\frac{1}{8}\left[\ln|2x+5| + 10(2x+5)^{-1} - \frac{25}{2}(2x+5)^{-2}\right] + C}$$

(6)



$\frac{18}{x} = 2x, \quad 2x^2 = 18, \quad x^2 = 9, \quad \underline{x=3}$ (since $x > 0$)

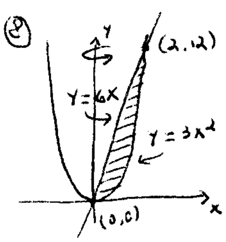
$$A = \int_1^3 \left(\frac{18}{x} - 2x\right) dx + \int_3^9 \left(2x - \frac{18}{x}\right) dx = \left[18\ln x - x^2\right]_1^3 + \left[x^2 - 18\ln x\right]_3^9$$

$$= (18\ln 3 - 9) - (0 - 1) + (81 - 18\ln 9) - (9 - 18\ln 3)$$

$$= 36\ln 3 - 18\ln 9 + 64 = 36\ln 3 - 18(2\ln 3) + 64 = \boxed{64}$$

(since $\ln 9 = \ln 3^2 = 2\ln 3$)

(8)

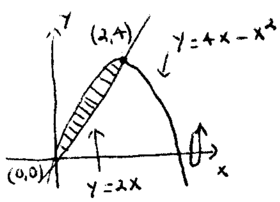


$3x^2 = 6x, \quad 3x^2 - 6x = 0, \quad 3x(x-2) = 0, \quad \underline{x=0} \text{ or } \underline{x=2}$

if $y = 3x^2$, $\underline{x = \sqrt{\frac{y}{3}}}$ and if $y = 6x$, $\underline{x = \frac{y}{6}}$

$$V = \boxed{\int_0^{12} \pi\left(\left(\sqrt{\frac{y}{3}}\right)^2 - \left(\frac{y}{6}\right)^2\right) dy} = \boxed{\pi\int_0^{12} \left(\frac{y}{3} - \frac{y^2}{36}\right) dy}$$

(9)



$$V = \int_0^2 \pi\left((4x - x^2)^2 - (2x)^2\right) dx = \pi\int_0^2 (16x^2 - 8x^3 + x^4 - 4x^2)\, dx$$

$$= \pi\int_0^2 (12x^2 - 8x^3 + x^4)\, dx = \pi\left[4x^3 - 2x^4 + \frac{x^5}{5}\right]_0^2$$

$$= \pi\left(32 - 32 + \frac{32}{5}\right) = \boxed{\frac{32\pi}{5}}$$

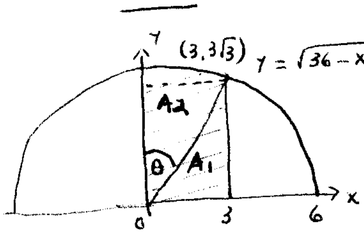
(10)

$$\int_{-3}^{3} (5 + 7t^9)\sqrt{36 - t^2}\, dt = \int_{-3}^{3} 5\sqrt{36 - t^2}\, dt + \int_{-3}^{3} 7t^9\sqrt{36 - t^2}\, dt$$

$$= 2\int_0^3 5\sqrt{36 - t^2}\, dt + 0 = 10\underline{\int_0^3 \sqrt{36 - t^2}\, dt}$$

since $f(t) = 5\sqrt{36 - t^2}$ is an even function and $g(t) = 7t^9\sqrt{36 - t^2}$ is an odd function:

$\underline{f(-t)} = 5\sqrt{36 - (-t)^2} = 5\sqrt{36 - t^2} = \underline{f(t)}$ and $\underline{g(-t)} = 7(-t)^9\sqrt{36 - (-t)^2} = -7t^9\sqrt{36 - t^2} = \underline{-g(t)}$



$$\underline{10\int_0^3 \sqrt{36 - t^2}\, dt} = 10\left[A_1 + A_2\right] = 10\left[\frac{1}{2}bh + \frac{1}{2}r^2\theta\right]$$

$$= 10\left[\frac{1}{2}\cdot 3 \cdot 3\sqrt{3} + \frac{1}{2}\cdot 6^2 \cdot \frac{\pi}{6}\right] = \boxed{45\sqrt{3} + 30\pi}$$

(where $\theta = \frac{\pi}{6}$ since $\sin\theta = \frac{3}{6} = \frac{1}{2}$)