① $\displaystyle\int \frac{1}{x^3+4x^2}\,dx$

$$\frac{1}{x^3+4x^2} = \frac{1}{x^2(x+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$$

$$1 = Ax(x+4) + B(x+4) + Cx^2$$

$x=0$: $1 = 4B$ $\quad B = 1/4$

$x=-4$: $1 = 16C$ $\quad C = 1/16$

x^2 coeff.: $0 = A + C$ $\quad A = -1/16$

$$\int\left(\frac{-1/16}{x} + \frac{1/4}{x^2} + \frac{1/16}{x+4}\right)dx = \boxed{-\frac{1}{16}\ln|x| - \frac{1}{4}x^{-1} + \frac{1}{16}\ln|x+4| + C}$$

$$= \frac{1}{16}\left[\ln|x+4| - \ln|x|\right] - \frac{1}{4}x^{-1} + C = \boxed{\frac{1}{16}\ln\left|\frac{x+4}{x}\right| - \frac{1}{4}x^{-1} + C}$$

② $\displaystyle\int e^{4x}\sin 2x\,dx$ $\qquad$ Let $u = e^{4x}$, $dv = \sin 2x\,dx$

$du = 4e^{4x}\,dx$, $v = -\frac{1}{2}\cos 2x$

$$\int e^{4x}\sin 2x\,dx = -\frac{1}{2}e^{4x}\cos 2x - (-2)\int e^{4x}\cos 2x\,dx$$

Let $u = e^{4x}$, $dv = \cos 2x\,dx$

$du = 4e^{4x}\,dx$, $v = \frac{1}{2}\sin 2x$

$$= -\frac{1}{2}e^{4x}\cos 2x + 2\left[\frac{1}{2}e^{4x}\sin 2x - 2\int e^{4x}\sin 2x\,dx\right]$$

$$= -\frac{1}{2}e^{4x}\cos 2x + e^{4x}\sin 2x - 4\int e^{4x}\sin 2x\,dx$$

$$5\int e^{4x}\sin 2x\,dx = -\frac{1}{2}e^{4x}\cos 2x + e^{4x}\sin 2x + C$$

$$\int e^{4x}\sin 2x\,dx = \boxed{-\frac{1}{10}e^{4x}\cos 2x + \frac{1}{5}e^{4x}\sin 2x + C}$$

OR use

u		dv
e^{4x}	$+$	$\sin 2x\,dx$
$4e^{4x}$	$-$	$-\frac{1}{2}\cos 2x$
$16e^{4x}$	$+$	$-\frac{1}{4}\sin 2x$

to get

$$\int e^{4x}\sin 2x\,dx = -\frac{1}{2}e^{4x}\cos 2x + e^{4x}\sin 2x - 4\int e^{4x}\sin 2x\,dx,$$

and then solve as above.

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③
$$\int \frac{3x^4-4x-12}{x^3-2x^2}\,dx = \int\left(3x+6 + \frac{12x^2-4x-12}{x^3-2x^2}\right)dx$$

$$\begin{array}{r} 3x+6 \\ x^3-2x^2\,\overline{)\,3x^4 \qquad\qquad -4x-12} \\ \underline{3x^4-6x^3} \\ 6x^3 \\ \underline{6x^3-12x^2} \\ 12x^2-4x-12 \end{array}$$

$$= \frac{3}{2}x^2 + 6x + \int\frac{12x^2-4x-12}{x^2(x-2)}\,dx$$

$$= \frac{3}{2}x^2 + 6x + \int\left(\frac{5}{x} + \frac{6}{x^2} + \frac{7}{x-2}\right)dx$$

$$\frac{12x^2-4x-12}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$12x^2-4x-12 = Ax(x-2) + B(x-2) + Cx^2$$

$$= \boxed{\frac{3}{2}x^2 + 6x + 5\ln|x| - \frac{6}{x} + 7\ln|x-2| + C}$$

$x=0$: $-12 = -2B$ $\quad B = 6$

$x=2$: $28 = 4C$ $\quad C = 7$

x^2 coeff.: $12 = A + C$ $\quad A = 5$

(4) $\displaystyle\int_1^{\sqrt{e}} \frac{\ln x}{x^3}\,dx$ Let $u = \ln x$, $dv = \frac{1}{x^3}\,dx = x^{-3}\,dx$

$du = \frac{1}{x}\,dx$, $v = -\frac{1}{2}x^{-2}$

$$= \left[(\ln x)\left(-\tfrac{1}{2}x^{-2}\right) - \left(-\tfrac{1}{2}\right)\int x^{-3}\,dx\right]_1^{\sqrt{e}} = \left[-\frac{\ln x}{2x^2} + \frac{1}{2}\cdot\left(-\tfrac{1}{2}x^{-2}\right)\right]_1^{\sqrt{e}}$$

$$= \left[-\frac{\ln x}{2x^2} - \frac{1}{4x^2}\right]_1^{\sqrt{e}} = \left[\frac{-2\ln x - 1}{4x^2}\right]_1^{\sqrt{e}} = \left(\frac{-2\cdot\frac{1}{2} - 1}{4e} - \frac{0-1}{4}\right) = -\frac{2}{4e} + \frac{1}{4} = \boxed{\frac{1}{4} - \frac{1}{2e}} = \boxed{\frac{e-2}{4e}}$$

(5) $\displaystyle\int_{1/4}^{\infty} \frac{15}{2x^2+5x+2}\,dx = \lim_{T\to\infty}\int_{1/4}^{T} \frac{15}{2x^2+5x+2}\,dx$

$\displaystyle\frac{15}{(2x+1)(x+2)} = \frac{A}{2x+1} + \frac{B}{x+2}$

$15 = A(x+2) + B(2x+1)$

$x = -2$: $15 = -3B$, $B = -5$

$x = -\frac{1}{2}$: $15 = \frac{3}{2}A$, $A = 10$

$$= \lim_{T\to\infty}\int_{1/4}^{T}\left(\frac{10}{2x+1} - \frac{5}{x+2}\right)dx$$

$$= \lim_{T\to\infty} 5\int_{1/4}^{T}\left(\frac{2}{2x+1} - \frac{1}{x+2}\right)dx = \lim_{T\to\infty} 5\Big[\ln(2x+1) - \ln(x+2)\Big]_{1/4}^{T}$$

$$= \lim_{T\to\infty} 5\left[\ln\left(\frac{2x+1}{x+2}\right)\right]_{1/4}^{T} = \lim_{T\to\infty} 5\left(\ln\left(\frac{2T+1}{T+2}\right) - \ln\left(\frac{3/2}{9/4}\right)\right)$$

$$= 5\left(\ln 2 - \ln\frac{2}{3}\right) = 5\left(\ln\frac{2}{2/3}\right) = \boxed{5\ln 3}$$

(since $\displaystyle\frac{2T+1}{T+2} = \frac{2+\frac{1}{T}}{1+\frac{2}{T}} \to \frac{2+0}{1+0} = 2$ as $T\to\infty$)

(6) $\displaystyle\int \cos\sqrt{x}\,dx$ Let $u = \sqrt{x}$, so $x = u^2$, $dx = 2u\,du$

$$= \int \cos u\cdot 2u\,du = 2\int u\cos u\,du$$

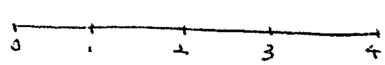
Let $u = u$, $dV = \cos u\,du$

$du = du$, $V = \sin u$

$$= 2\left[u\sin u - \int \sin u\,du\right]$$

$$= 2\left[u\sin u - (-\cos u)\right] + C = \boxed{2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C}$$

(8) $S = \displaystyle\int_0^4 \sqrt{1+4x^2}\,dx$, $n = 4$



$\Delta x = \frac{4-0}{4} = 1$

A) TRAPEZOIDAL RULE

$$S \approx \boxed{\frac{1}{2}\left[1 + 2\sqrt{5} + 2\sqrt{17} + 2\sqrt{37} + \sqrt{65}\right]} \approx 16.973$$

B) SIMPSON'S RULE

$$S \approx \boxed{\frac{1}{3}\left[1 + 4\sqrt{5} + 2\sqrt{17} + 4\sqrt{37} + \sqrt{65}\right]} \approx 16.861$$

(EXACT VALUE $= 2\sqrt{65} + \frac{1}{4}\ln(8+\sqrt{65}) \approx 16.819$)

(7) $f(T) = \dfrac{1}{2\sqrt{2T+1}}$, $[0,4]$

A) $P(T < \frac{3}{2}) = \displaystyle\int_0^{3/2} \frac{1}{2\sqrt{2T+1}}\, dT$ LET $u = 2T+1$, $du = 2\,dT$ IF $T=0$, $u=1$; $T=3/2$, $u=4$

$= \dfrac{1}{2}\displaystyle\int_0^{3/2} \frac{1}{2\sqrt{2T+1}}\cdot 2\, dT = \frac{1}{2}\int_1^4 \frac{1}{2\sqrt{u}}\, du = \frac{1}{4}\int_1^4 u^{-1/2}\, du = \frac{1}{4}\left[2u^{1/2}\right]_1^4 = \frac{1}{2}(2-1) = \boxed{\frac{1}{2}}$

OR $P(T < \frac{3}{2}) = \displaystyle\int_0^{3/2} \frac{1}{2\sqrt{2T+1}}\, dT$ LET $u = \sqrt{2T+1}$, $T = \frac{1}{2}(u^2-1)$, $dT = \frac{1}{2}(2u)\,du$ IF $T=0$, $u=1$; $T=3/2$, $u=2$

$= \displaystyle\int_1^2 \frac{1}{2\cancel{u}}\cdot \cancel{u}\, du = \frac{1}{2}\int_1^2 1\, du = \frac{1}{2}\left[u\right]_1^2 = \frac{1}{2}(2-1) = \boxed{\frac{1}{2}}$

B) $\mu = E(T) = \displaystyle\int_0^4 T\cdot \frac{1}{2\sqrt{2T+1}}\, dT$ LET $u = \sqrt{2T+1}$, $T = \frac{1}{2}(u^2-1)$, $dT = u\, du$ IF $T=0$, $u=1$; $T=4$, $u=3$

$= \displaystyle\int_1^3 \frac{1}{2}(u^2-1)\cdot \frac{1}{2\cancel{u}}\cdot \cancel{u}\, du = \frac{1}{4}\int_1^3 (u^2-1)\, du = \frac{1}{4}\left[\frac{u^3}{3} - u\right]_1^3$

$= \frac{1}{4}\left((9-3) - (\frac{1}{3} - 1)\right) = \frac{1}{4}\cdot\frac{20}{3} = \boxed{\frac{5}{3} \text{ DAYS}}$

OR $\mu = E(T) = \displaystyle\int_0^4 T\cdot \frac{1}{2\sqrt{2T+1}}\, dT$ LET $u = 2T+1$, $T = \frac{1}{2}(u-1)$, $dT = \frac{1}{2}\, du$ IF $T=0$, $u=1$; $T=4$, $u=9$

$= \displaystyle\int_1^9 \frac{1}{2}(u-1)\cdot \frac{1}{2\sqrt{u}}\cdot \frac{1}{2}\, du = \frac{1}{8}\int_1^9 \frac{u-1}{\sqrt{u}}\, du = \frac{1}{8}\int_1^9 \left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right) du$

$= \frac{1}{8}\displaystyle\int_1^9 (u^{1/2} - u^{-1/2})\, du = \frac{1}{8}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9 = \frac{1}{8}\left(\left(\frac{2}{3}\cdot 27 - 2\cdot 3\right) - \left(\frac{2}{3} - 2\right)\right)$

$= \frac{1}{8}\left(\frac{52}{3} - 4\right) = \frac{1}{8}\cdot\frac{40}{3} = \boxed{\frac{5}{3} \text{ DAYS}}$

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(9) $\displaystyle\int_0^\infty \frac{1}{x+\sqrt{x}}\, dx = \int_0^1 \frac{1}{x+\sqrt{x}}\, dx + \int_1^\infty \frac{1}{x+\sqrt{x}}\, dx$

A) $\displaystyle\int_1^\infty \frac{1}{x+\sqrt{x}}\, dx = \lim_{T\to\infty}\int_1^T \frac{1}{x+\sqrt{x}}\, dx = \lim_{T\to\infty}\int_1^T \frac{1}{\sqrt{x}(\sqrt{x}+1)}\, dx$ LET $u = \sqrt{x}+1$, $du = \frac{1}{2\sqrt{x}}\, dx$

IF $x=1$, $u=2$; $x=T$, $u=\sqrt{T}+1$

$= \displaystyle\lim_{T\to\infty} 2\int_1^T \frac{1}{\sqrt{x}+1}\ \frac{1}{2\sqrt{x}}\, dx$

$= \displaystyle\lim_{T\to\infty} 2\int_2^{\sqrt{T}+1} \frac{1}{u}\, du = \lim_{T\to\infty} 2\left[\ln u\right]_2^{\sqrt{T}+1} = \lim_{T\to\infty} 2\left(\ln(\sqrt{T}+1) - \ln 2\right) = \infty,$

SO $\displaystyle\int_0^\infty \frac{1}{x+\sqrt{x}}\, dx$ $\boxed{\text{DIVERGES}}$

REMARK THE ORIGINAL INTEGRAL HAD TO BE SPLIT UP BECAUSE IT WAS IMPROPER FOR 2 REASONS, THE INFINITE LIMIT AND THE DISCONTINUITY AT 0.

(32) $\int_0^1 \frac{1}{x+1}\,dx$, $n=4$

a) TRAPEZOIDAL RULE $\quad f(x)=(x+1)^{-1} \quad f'(x)=-(x+1)^{-2} \quad f''(x)=2(x+1)^{-3}=\frac{2}{(x+1)^3}$

so $\max\limits_{0\le x\le 1}|f''(x)| = \max\limits_{0\le x\le 1}\frac{2}{(x+1)^3}=\frac{2}{(0+1)^3}=2$ (since $y=\frac{2}{(x+1)^3}$ is decreasing on $[0,1]$)

$$|E|\le \frac{(b-a)^3}{12n^2}\left[\max_{a\le x\le b}|f''(x)|\right]=\frac{(1-0)^3}{12\cdot 4^2}\cdot 2=\frac{2}{12\cdot 16}=\boxed{\frac{1}{96}}\approx .0104$$

b) SIMPSON'S RULE $\quad f'''(x)=-6(x+1)^{-4} \quad f^{(4)}(x)=24(x+1)^{-5}=\frac{24}{(x+1)^5}$

so $\max\limits_{0\le x\le 1}|f^{(4)}(x)|=\max\limits_{0\le x\le 1}\frac{24}{(x+1)^5}=\frac{24}{(0+1)^5}=24$ (since $y=\frac{24}{(x+1)^5}$ is decreasing on $[0,1]$)

$$|E|\le\frac{(b-a)^5}{180n^4}\left[\max_{a\le x\le b}|f^{(4)}(x)|\right]=\frac{(1-0)^5}{180\cdot 4^4}\cdot 24=\frac{24}{180\cdot 4\cdot 64}=\frac{1}{30\cdot 64}=\boxed{\frac{1}{1920}}\approx .0005$$

(36) $\int_1^3 \frac{1}{x}\,dx$; find n so that $|E|<.0001$

a) TRAPEZOIDAL RULE $\quad f(x)=x^{-1} \quad f'(x)=-x^{-2} \quad f''(x)=2x^{-3}=\frac{2}{x^3}$

so $\max\limits_{1\le x\le 3}|f''(x)|=\max\limits_{1\le x\le 3}\frac{2}{x^3}=\frac{2}{1^3}=2$ (since $y=\frac{2}{x^3}$ is decreasing on $[1,3]$)

$$|E|\le\frac{(b-a)^3}{12n^2}\left[\max_{1\le x\le 3}|f''(x)|\right]=\frac{(3-1)^3}{12n^2}\cdot 2=\frac{8\cdot 2}{12n^2}=\frac{4}{3n^2}<.0001 \text{ iff}$$

$$\frac{4}{3}(10{,}000)<n^2 \text{ iff } n>\sqrt{\frac{40{,}000}{3}}\approx 115.47$$

so $n=\boxed{116}$ is the smallest value of n which will guarantee that $|E|<.0001$

b) SIMPSON'S RULE $\quad f'''(x)=-6x^{-4} \quad f^{(4)}(x)=24x^{-5}=\frac{24}{x^5}$ so

$\max\limits_{1\le x\le 3}|f^{(4)}(x)|=\max\limits_{1\le x\le 3}\frac{24}{x^5}=\frac{24}{1^5}=24$ (since $y=\frac{24}{x^5}$ is decreasing on $[1,3]$)

$$|E|\le\frac{(b-a)^5}{180n^4}\left[\max_{a\le x\le b}|f^{(4)}(x)|\right]=\frac{(3-1)^5}{180n^4}\cdot 24=\frac{32(24)}{180n^4}=\frac{64}{15n^4}<.0001 \text{ iff}$$

$$\frac{64}{15}(10{,}000)<n^4 \text{ iff } n^4>\frac{128{,}000}{3} \text{ iff } n>\sqrt[4]{\frac{128{,}000}{3}}\approx 14.37$$

so $n=\boxed{16}$ is the smallest value of n which will guarantee that $|E|<.0001$

(since n must be even in Simpson's Rule).