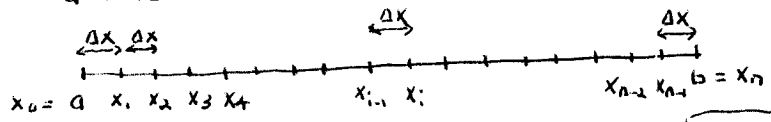
Sec. 5.6 and 6.5 – Approximating Definite Integrals

To approximate $\int_a^b f(x)\,dx$, we can first divide $[a,b]$ into n equal subintervals by choosing equally spaced numbers $x_0, x_1, x_2, \dots, x_n$ with

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b:$$



Each subinterval has length given by $\boxed{\Delta x = \frac{b-a}{n}}$.

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① Midpoint Rule

If $c_1, \dots, c_n$ are the midpoints of the subintervals (so $c_i = \frac{x_{i-1} + x_i}{2}$ for each i),

$$\int_a^b f(x)\,dx \approx \boxed{f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x}$$

$$= \boxed{\left[f(c_1) + f(c_2) + \cdots + f(c_n)\right]\Delta x}$$

② Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \boxed{\frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]}$$

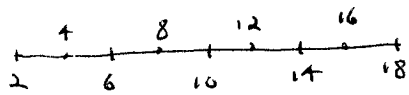
Coefficients: $1, 2, 2, \dots, 2, 1$

③ Simpson's Rule (n even)

$$\int_a^b f(x)\,dx \approx \boxed{\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]}$$

Coefficients: $1, 4, 2, 4, 2, 4, \dots, 2, 4, 1$

Ex A) $\int_2^{18} \frac{1}{x+2}\,dx$, $n=4$



$\Delta x = \frac{18-2}{4} = 4$

1) MIDPOINT RULE

$$\int_2^{18} \frac{1}{x+2}\,dx \approx \frac{1}{6}\cdot 4 + \frac{1}{10}\cdot 4 + \frac{1}{14}\cdot 4 + \frac{1}{18}\cdot 4 \approx 1.5746$$

2) TRAPEZOIDAL RULE

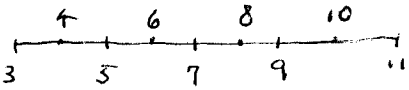
$$\int_2^{18} \frac{1}{x+2}\,dx \approx \frac{4}{2}\left[\frac{1}{4} + 2\cdot\frac{1}{8} + 2\cdot\frac{1}{12} + 2\cdot\frac{1}{16} + \frac{1}{20}\right] \approx 1.6833$$

3) SIMPSON'S RULE

$$\int_2^{18} \frac{1}{x+2}\,dx \approx \frac{4}{3}\left[\frac{1}{4} + 4\cdot\frac{1}{8} + 2\cdot\frac{1}{12} + 4\cdot\frac{1}{16} + \frac{1}{20}\right] \approx 1.6222$$

EXACT VALUE = $\ln 5 = 1.6094\ldots$

B) $\int_3^{11} \frac{x}{\ln x}\,dx$, $n=4$



$\Delta x = \frac{11-3}{4} = 2$

1) MIDPOINT RULE

$$\int_3^{11} \frac{x}{\ln x}\,dx \approx \frac{4}{\ln 4}\cdot 2 + \frac{6}{\ln 6}\cdot 2 + \frac{8}{\ln 8}\cdot 2 + \frac{10}{\ln 10}\cdot 2 \approx 28.848$$

2) TRAPEZOIDAL RULE

$$\int_3^{11} \frac{x}{\ln x}\,dx \approx \frac{2}{2}\left[\frac{3}{\ln 3} + 2\cdot\frac{5}{\ln 5} + 2\cdot\frac{7}{\ln 7} + 2\cdot\frac{9}{\ln 9} + \frac{11}{\ln 11}\right] \approx 28.918$$

3) SIMPSON'S RULE

$$\int_3^{11} \frac{x}{\ln x}\,dx \approx \frac{2}{3}\left[\frac{3}{\ln 3} + 4\cdot\frac{5}{\ln 5} + 2\cdot\frac{7}{\ln 7} + 4\cdot\frac{9}{\ln 9} + \frac{11}{\ln 11}\right] \approx 28.882$$

ACTUAL VALUE = $28.870\ldots$