① A) $\displaystyle\int \frac{12x^2}{x^3+5}\,dx$ LET $u = x^3+5$, $du = 3x^2\,dx$

$$= \frac{12}{3}\int \frac{3x^2}{x^3+5}\,dx = \boxed{4\ln|x^3+5| + C}$$

B) $\displaystyle\int 9x\sqrt{x^2+4}\,dx$ LET $u = x^2+4$, $du = 2x\,dx$

$$= \frac{9}{2}\int \sqrt{x^2+4}\cdot 2x\,dx = \frac{9}{2}\int \sqrt{u}\,du = \frac{9}{2}\int u^{1/2}\,du = \frac{9}{2}\left(\frac{2}{3}u^{3/2}\right) + C = \boxed{3(x^2+4)^{3/2} + C}$$

C) $\displaystyle\int \frac{6}{x(\ln 3x)^2}\,dx$ LET $u = \ln 3x$, $du = \frac{3}{3x}\,dx = \frac{1}{x}\,dx$

$$= 6\int \frac{1}{(\ln 3x)^2}\cdot\frac{1}{x}\,dx = 6\int \frac{1}{u^2}\,du = 6\int u^{-2}\,du = 6(-u^{-1}) + C = \boxed{-6(\ln 3x)^{-1} + C} = \boxed{-\frac{6}{\ln 3x} + C}$$

D) $\displaystyle\int \frac{80e^{2x}}{(5e^{2x}+1)^3}\,dx$ LET $u = 5e^{2x}+1$, $du = 10e^{2x}\,dx$

$$= \frac{80}{10}\int \frac{1}{(5e^{2x}+1)^3}\cdot 10e^{2x}\,dx = 8\int \frac{1}{u^3}\,du = 8\int u^{-3}\,du = 8\left(-\frac{1}{2}u^{-2}\right) + C = \boxed{-4(5e^{2x}+1)^{-2} + C}$$

② $f(x) = (e^{2x}+1)^4$ $f'(x) = 4(e^{2x}+1)^3\cdot(e^{2x}\cdot 2) = 8(e^{2x}+1)^3\cdot e^{2x}$

$\underline{m} = f'(0) = 8\cdot 2^3\cdot 1 = \underline{64}$ $\boxed{y - 16 = 64(x-0)}$ OR $\boxed{y = 64x + 16}$

③ $f'(x) = \dfrac{5x^3-6}{x^4}$ FOR $x > 0$; PASSES THROUGH $(1,10)$

$$f(x) = \int\left(\frac{5x^3-6}{x^4}\right)dx = \int\left(\frac{5x^3}{x^4} - \frac{6}{x^4}\right)dx = \int(5x^{-1} - 6x^{-4})\,dx = \underline{5\ln|x| + 2x^{-3} + C}$$

$f(1) = 5\ln 1 + 2 + C = 0 + 2 + C = 10$ SO $\underline{C = 8}$ AND $\boxed{f(x) = 5\ln x + 2x^{-3} + 8}$

(CAN DROP ABS. VALUES, SINCE $x > 0$)

④ $x^5y^2 - e^{y^4} - \ln 8y = x^3 + 9x$

$$\frac{d}{dx}\left(x^5y^2 - e^{y^4} - \ln 8y\right) = \frac{d}{dx}(x^3+9x)$$

$$x^5\cdot 2yy' + 5x^4y^2 - e^{y^4}\cdot 4y^3y' - \frac{\cancel{8}y'}{\cancel{8}y} = 3x^2 + 9$$

$$2x^5yy' - 4y^3e^{y^4}y' - \frac{1}{y}y' = 3x^2 + 9 - 5x^4y^2$$

$$\left(2x^5y - 4y^3e^{y^4} - \frac{1}{y}\right)y' = 3x^2 + 9 - 5x^4y^2$$

$$y' = \boxed{\frac{3x^2 + 9 - 5x^4y^2}{2x^5y - 4y^3e^{y^4} - \frac{1}{y}}}$$

⑦ $\displaystyle\int 36\sqrt{x}\,(x^{3/2}+9)^5\,dx$ LET $u = x^{3/2}+9$, $du = \frac{3}{2}x^{1/2}\,dx$

$$= 36\cdot\frac{2}{3}\int (x^{3/2}+9)^5\cdot\frac{3}{2}x^{1/2}\,dx = 24\int u^5\,du = 24\left(\frac{u^6}{6}\right) + C = \boxed{4(x^{3/2}+9)^6 + C}$$

⑤ $y = \left(e^{4/x} + 5^{\sqrt{x}}\right)^{\log_2 x}$

1) $\ln y = \ln\left(e^{4/x} + 5^{\sqrt{x}}\right)^{\log_2 x} = \log_2 x \,\ln\left(e^{4/x} + 5^{\sqrt{x}}\right)$

2) $\dfrac{y'}{y} = \log_2 x \cdot \left(\dfrac{e^{4/x}\left(-\frac{4}{x^2}\right) + 5^{\sqrt{x}} \cdot \ln 5 \cdot \frac{1}{2\sqrt{x}}}{e^{4/x} + 5^{\sqrt{x}}} \right) + \dfrac{1}{x \ln 2} \cdot \ln\left(e^{4/x} + 5^{\sqrt{x}}\right)$

3) $y' = \boxed{\left(e^{4/x} + 5^{\sqrt{x}}\right)^{\log_2 x} \left[\log_2 x \cdot \left(\dfrac{e^{4/x}\left(-\frac{4}{x^2}\right) + 5^{\sqrt{x}} \cdot \ln 5 \cdot \frac{1}{2\sqrt{x}}}{e^{4/x} + 5^{\sqrt{x}}} \right) + \dfrac{1}{x \ln 2} \cdot \ln\left(e^{4/x} + 5^{\sqrt{x}}\right) \right]}$

⑥ $y = Ce^{kt} = 8e^{kt}$

WHEN $t = 11$, $y = 5$: $8e^{11k} = 5$, $e^{11k} = \frac{5}{8}$, $\ln e^{11k} = \ln\frac{5}{8}$, $11k = \ln\frac{5}{8}$, $k = \frac{1}{11}\ln\frac{5}{8}$

$y = 8e^{\left(\frac{1}{11}\ln\frac{5}{8}\right)t} = 8\left(e^{\ln\frac{5}{8}}\right)^{\frac{t}{11}} = 8\left(\frac{5}{8}\right)^{t/11}$ (OR $e^k = \left(\frac{5}{8}\right)^{\frac{1}{11}}$)

WHEN $y = .60(8)$, $\cancel{8}\left(\frac{5}{8}\right)^{t/11} = .6(\cancel{8})$, $\left(\frac{5}{8}\right)^{\frac{t}{11}} = .6$,

$\ln\left(\frac{5}{8}\right)^{t/11} = \ln .6$, $\frac{t}{11}\ln\frac{5}{8} = \ln .6$, $t = \boxed{\dfrac{11\ln .6}{\ln\frac{5}{8}} \text{ YR}}$

⑧ $f(x) = \dfrac{(\ln x)^3}{x}$ $f'(x) = \dfrac{x \cdot 3(\ln x)^2 \cdot \frac{1}{x} - (\ln x)^3 \cdot 1}{x^2} = \dfrac{3(\ln x)^2 - (\ln x)^3}{x^2} = \dfrac{(\ln x)^2(3 - \ln x)}{x^2}$

$f'(x) = 0$ IF $\ln x = 0$ OR $\ln x = 3$, SO $\boxed{x = 1}$ AND $\boxed{x = e^3}$ ARE THE CRITICAL NUMBERS.

$+$ $+$ $-$ f' (SAME SIGNS AS $3 - \ln x$)

0 1 e^3

$f'(e^{-1}) = 4e^2$ $f'(e) = \frac{2}{e^2}$ $f'(e^4) = \frac{4^2(-1)}{e^8}$

$\boxed{f(e^3) = \frac{27}{e^3} \text{ IS A REL. MAX.}}$ (NO REL. EXTREMUM AT 1)

⑨ $T = S + Ce^{kt}$

① $t=0$ GIVES $205 = S + C$

② $t=8$ GIVES $175 = S + Ce^{8k}$

③ $t=16$ GIVES $151 = S + Ce^{16k}$

① − ② GIVES $30 = C - Ce^{8k} = C(1 - e^{8k})$

② − ③ GIVES $24 = Ce^{8k} - Ce^{16k} = Ce^{8k}(1 - e^{8k}) = \left[C(1-e^{8k})\right]e^{8k} = 30e^{8k}$

SO $e^{8k} = \frac{24}{30} = \frac{4}{5}$

THEN $30 = C\left(1 - \frac{4}{5}\right) = \frac{C}{5}$ SO $C = 150$ AND $S = 205 - C = \boxed{55^\circ}$

OR ① GIVES $C = 205 - S$, SO ② GIVES $e^{8k} = \dfrac{175 - S}{205 - S}$ AND ③ GIVES $e^{16k} = \dfrac{151 - S}{205 - S}$

THEN $\dfrac{151 - S}{205 - S} = (e^{8k})^2 = \left(\dfrac{175 - S}{205 - S}\right)^2$, SO MULTIPLYING BY $(205 - S)^2$ GIVES

$(151 - S)(205 - S) = (175 - S)^2$, $151(205) - 356S + \cancel{S^2} = 175^2 - 350S + \cancel{S^2}$,

$6S = (151)(205) - 175^2 = 330$, $S = \boxed{55^\circ}$