① $f(x) = \int \frac{4x+3}{x^2}\,dx = \int \left(\frac{4x}{x^2} + \frac{3}{x^2}\right)dx = \int\left(\frac{4}{x} + 3x^{-2}\right)dx = \underline{4\ln|x| - \frac{3}{x} + C}$

$f(1) = 4\ln 1 - 3 + C = 0 - 3 + C = 5$, so $\underline{C = 8}$ and $\boxed{f(x) = 4\ln x - \frac{3}{x} + 8}$ for $x > 0$

② $\int \frac{24x^2}{(x^3+1)^3}\,dx$ Let $\underline{u = x^3+1}$, $du = 3x^2\,dx$

$= \frac{24}{3}\int \frac{1}{(x^3+1)^3}\cdot 3x^2\,dx = 8\int \frac{1}{u^3}\,du = 8\int u^{-3}\,du = 8\left(-\frac{1}{2}u^{-2}\right) + C = \boxed{-4(x^3+1)^{-2} + C}$

③ $\int \frac{5x-20}{x^2-8x+2}\,dx$ Let $\underline{u = x^2-8x+2}$, $du = (2x-8)\,dx = 2(x-4)\,dx$

$= \int \frac{5(x-4)}{x^2-8x+2}\,dx = \frac{5}{2}\int \frac{2(x-4)}{x^2-8x+2}\,dx = \boxed{\frac{5}{2}\ln|x^2-8x+2| + C}$ (using the log rule)

④ $\int \frac{12}{\sqrt{x}(\sqrt{x}+5)^4}\,dx$ Let $\underline{u = \sqrt{x}+5}$, $du = \frac{1}{2\sqrt{x}}\,dx$

$= 12\cdot 2\int \frac{1}{(\sqrt{x}+5)^4}\cdot\frac{1}{2\sqrt{x}}\,dx = 24\int \frac{1}{u^4}\,du = 24\int u^{-4}\,du = 24\left(-\frac{1}{3}u^{-3}\right) + C = \boxed{-8(\sqrt{x}+5)^{-3} + C}$

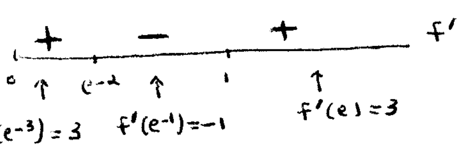
⑤ $\int \frac{15e^{2x}}{\sqrt{3e^{2x}+4}}\,dx$ Let $\underline{u = 3e^{2x}+4}$, $du = 6e^{2x}\,dx$

$= \frac{15}{6}\int \frac{1}{\sqrt{3e^{2x}+4}}\cdot 6e^{2x}\,dx = \frac{5}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{5}{2}\int u^{-1/2}\,du = \frac{5}{2}\left[2u^{1/2}\right] + C = \boxed{5\sqrt{3e^{2x}+4} + C}$

⑥ $f(x) = x(\ln x)^2$

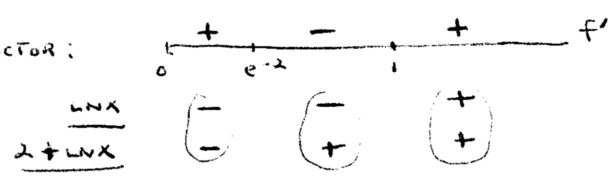
$f'(x) = \cancel{x}\left(2\ln x\cdot\frac{1}{\cancel{x}}\right) + 1\cdot(\ln x)^2 = 2\ln x + (\ln x)^2 = \ln x\,(2+\ln x) = 0$

if 1) $\ln x = 0$, so $\underline{x = 1}$ or 2) $\ln x = -2$, so $\underline{x = e^{-2}}$



$\boxed{\begin{array}{l} f(e^{-2}) = 4e^{-2} = \frac{4}{e^2} \text{ is a rel. max.} \\ f(1) = 0 \text{ is a rel. min.}\end{array}}$

(or) use the signs of each factor:



⑦ At US Bank, $P = 1000e^{.06t}$ so $\underline{P = 1000e^{.3}}$ when $t = 5$

At Wells Fargo, $P = 1000\left(1+\frac{r}{12}\right)^{12t}$ so $\underline{P = 1000\left(1+\frac{r}{12}\right)^{60}}$ when $t = 5$

Then $\cancel{1000}\left(1+\frac{r}{12}\right)^{60} = \cancel{1000}e^{.3} \Rightarrow \left(1+\frac{r}{12}\right)^{60} = e^{.3} \Longrightarrow 1 + \frac{r}{12} = (e^{.3})^{\frac{1}{60}} = e^{.005}$,

so $\frac{r}{12} = e^{.005} - 1$ and $r = \boxed{12(e^{.005}-1)} \approx 6.015\%$

OR use effective annual yield:

$i = \underline{e^{.06} - 1}$ at US Bank, and $i = \underline{\left(1+\frac{r}{12}\right)^{12} - 1}$ at Wells Fargo,

so $e^{.06} - 1 = \left(1+\frac{r}{12}\right)^{12} - 1 \Rightarrow e^{.06} = \left(1+\frac{r}{12}\right)^{12} \Longrightarrow e^{.005} = 1 + \frac{r}{12}$,

so $\frac{r}{12} = e^{.005} - 1$ and $r = \boxed{12(e^{.005}-1)} \approx 6.015\%$

(8) $y = (2x)^{\sqrt{x}} (\ln x)^x$

1) $\underline{\ln y} = \ln (2x)^{\sqrt{x}} + \ln (\ln x)^x = \underline{\sqrt{x} \ln 2x + x \ln (\ln x)}$

2) $\underline{\dfrac{y'}{y}} = \sqrt{x} \cdot \dfrac{2}{2x} + \dfrac{1}{2\sqrt{x}} \ln 2x + x \cdot \dfrac{\frac{1}{x}}{\ln x} + 1 \cdot \ln(\ln x)$

$= \dfrac{\sqrt{x}}{x} + \dfrac{\ln 2x}{2\sqrt{x}} + \dfrac{1}{\ln x} + \ln(\ln x)$

3) $y' = \boxed{(2x)^{\sqrt{x}} (\ln x)^x \left[\dfrac{1}{\sqrt{x}} + \dfrac{\ln 2x}{2\sqrt{x}} + \dfrac{1}{\ln x} + \ln(\ln x)\right]}$ SINCE $\dfrac{\sqrt{x}}{x} = \dfrac{\sqrt{x}}{(\sqrt{x})^2} = \dfrac{1}{\sqrt{x}}$

(9) 1) $\underline{y = Ce^{kt} = 7e^{kt}}$

2) WHEN $t=5$, $y=8$; SO $7e^{5k} = 8$, $e^{5k} = \frac{8}{7}$, $\underline{e^k = \left(\frac{8}{7}\right)^{1/5}}$ (OR $\underline{k = \frac{1}{5}\ln\frac{8}{7}}$)

AND $y = 7(e^k)^t = 7\left(\left(\frac{8}{7}\right)^{1/5}\right)^t = \underline{7\left(\frac{8}{7}\right)^{t/5}}$ (OR $y = \underline{7e^{(\frac{1}{5}\ln\frac{8}{7})t}}$)

3) WHEN $y = 7 + .4(7) = 1.4(7)$,

$\cancel{7}\left(\frac{8}{7}\right)^{t/5} = 1.4(\cancel{7})$, $\left(\frac{8}{7}\right)^{t/5} = 1.4$, $\ln\left(\frac{8}{7}\right)^{t/5} = \ln 1.4$, $\frac{t}{5}\ln\frac{8}{7} = \ln 1.4$, $t = \boxed{\dfrac{5\ln 1.4}{\ln 8/7} \text{ HR}}$

(10) 1) $\underline{v(t)} = \int a(t)\,dt = \int -5\,dt = -5t + C = \underline{-5t+15}$ SINCE $C = v(0) = 15$

2) $\underline{s(t)} = \int v(t)\,dt = \int(-5t+15)\,dt = -\frac{5}{2}t^2 + 15t + D = \underline{-\frac{5}{2}t^2 + 15t + 40}$ SINCE $D = s(0) = 40$

3) WHEN $s(t) = 0$, $-\frac{5}{2}t^2 + 15t + 40 = 0$, $-5t^2 + 30t + 80 = 0$, $t^2 - 6t - 16 = 0$, $(t-8)(t+2) = 0$,
$\underline{t = 8}$ OR $\cancel{t = -2}$

4) $\underline{v(8)} = -40 + 15 = \boxed{-25 \text{ FT/SEC}}$

(11) 1) $\underline{T = S + Ce^{kt}} = \underline{70 + Ce^{kt}}$

2) $t=0$ GIVES $170 = 70 + C \cdot 1$ SO $\underline{C = 100}$ AND $\underline{T = 70 + 100e^{kt}}$

3) $t=5$ GIVES $150 = 70 + 100e^{5k}$, $80 = 100e^{5k}$, $e^{5k} = \frac{8}{10} = \frac{4}{5}$, $\underline{e^k = \left(\frac{4}{5}\right)^{1/5}}$

SO $\underline{T = 70 + 100\left(\frac{4}{5}\right)^{t/5}}$

4) IF $T = 130$, $130 = 70 + 100\left(\frac{4}{5}\right)^{t/5}$, $60 = 100\left(\frac{4}{5}\right)^{t/5}$, $\frac{3}{5} = \left(\frac{4}{5}\right)^{t/5}$, $\ln\left(\frac{4}{5}\right)^{t/5} = \ln\frac{3}{5}$,

$\frac{t}{5}\ln\frac{4}{5} = \ln\frac{3}{5}$, $t = \boxed{\dfrac{5\ln\frac{3}{5}}{\ln\frac{4}{5}} \text{ MIN}} = \boxed{\dfrac{5\ln .6}{\ln .8} \text{ MIN}}$

(12) $\displaystyle\int \frac{\sin\theta}{\sin\theta + \cos\theta}\,d\theta = \frac{1}{2}\int \frac{2\sin\theta}{\sin\theta + \cos\theta}\,d\theta = \frac{1}{2}\int \frac{(\sin\theta + \cos\theta) - (\cos\theta - \sin\theta)}{\sin\theta + \cos\theta}\,d\theta$

$\displaystyle = \frac{1}{2}\int\left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta} - \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\right)d\theta$

$\displaystyle = \frac{1}{2}\int\left(1 - \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\right)d\theta = \boxed{\frac{1}{2}\left[\theta - \ln|\sin\theta + \cos\theta|\right] + C}$

(USING $u = \sin\theta + \cos\theta$, $du = (\cos\theta - \sin\theta)\,d\theta$ TO INTEGRATE THE 2ND TERM)

(OR) MULTIPLY BY $\cos\theta - \sin\theta$ ON THE TOP AND BOTTOM TO GET

$\displaystyle\int \frac{\sin\theta(\cos\theta - \sin\theta)}{(\sin\theta + \cos\theta)(\cos\theta - \sin\theta)}\,d\theta = \int \frac{\sin\theta\cos\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta}\,d\theta = \int \frac{\frac{1}{2}\sin 2\theta - \frac{1}{2}(1 - \cos 2\theta)}{\cos 2\theta}\,d\theta$

$\displaystyle = \frac{1}{2}\int \frac{\sin 2\theta - 1 + \cos 2\theta}{\cos 2\theta}\,d\theta = \frac{1}{2}\int(\tan 2\theta - \sec 2\theta + 1)\,d\theta$

$= \boxed{\frac{1}{2}\left[-\frac{1}{2}\ln|\cos 2\theta| - \frac{1}{2}\ln|\sec 2\theta + \tan 2\theta| + \theta\right] + C}$