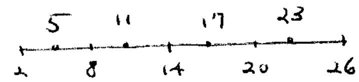
ANSWERS – TEST 3 REVIEW PROBLEMS

① A) MIDPOINT RULE

$$\int_2^{26} \frac{1}{\ln x}\,dx \approx \boxed{\frac{1}{\ln 5}\cdot 6 + \frac{1}{\ln 11}\cdot 6 + \frac{1}{\ln 17}\cdot 6 + \frac{1}{\ln 23}\cdot 6}$$



$$\underline{\Delta x} = \frac{26-2}{4} = \underline{6}$$

B) TRAPEZOIDAL RULE

$$\int_2^{26} \frac{1}{\ln x}\,dx \approx \boxed{\frac{6}{2}\left[\frac{1}{\ln 2} + 2\cdot\frac{1}{\ln 8} + 2\cdot\frac{1}{\ln 14} + 2\cdot\frac{1}{\ln 20} + \frac{1}{\ln 26}\right]}$$

② $$\int \frac{2x^2-5x+20}{x(x-2)^2}\,dx \qquad \frac{2x^2-5x+20}{x(x-2)^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$2x^2-5x+20 = A(x-2)^2 + Bx(x-2) + Cx$$

$\underline{x=0}$: $20 = 4A$ $\quad \underline{A=5}$

$\underline{x=2}$: $18 = 2C$ $\quad \underline{C=9}$

$\underline{x^2 \text{ coeff.}}$: $2 = A+B = 5+B$ $\quad \underline{B=-3}$

$$\int\left(\frac{5}{x} - \frac{3}{x-2} + \frac{9}{(x-2)^2}\right)dx = 5\ln|x| - 3\ln|x-2| + 9\int (x-2)^{-2}\,dx$$

LET $\underline{u = x-2}$, $du = dx$

$$= \boxed{5\ln|x| - 3\ln|x-2| - 9(x-2)^{-1} + C}$$

③ A) $$\mu = \int_1^5 x\cdot\frac{25}{12x^3}\,dx = \frac{25}{12}\int_1^5 \frac{1}{x^2}\,dx = \frac{25}{12}\left[-\frac{1}{x}\right]_1^5 = \frac{25}{12}\left(-\frac{1}{5}-(-1)\right) = \frac{25}{12}\cdot\frac{4}{5} = \boxed{\frac{5}{3}}$$

B) $$V(x) = \left(\int_1^5 x^2\cdot\frac{25}{12x^3}\,dx\right) - \mu^2 = \frac{25}{12}\int_1^5 \frac{1}{x}\,dx - \frac{25}{9} = \frac{25}{12}\Big[\ln x\Big]_1^5 - \frac{25}{9}$$

$$= \frac{25}{12}(\ln 5 - \underset{0}{\ln 1}) - \frac{25}{9} = \boxed{\frac{25}{12}\ln 5 - \frac{25}{9}} = \boxed{\frac{25}{36}(3\ln 5 - 4)}$$

④ A) $$\int_0^{1/e} \frac{24}{x(\ln x)^4}\,dx = \lim_{T\to 0^+}\int_T^{1/e}\frac{24}{x(\ln x)^4}\,dx$$

LET $\underline{u = \ln x}$, $du = \frac{1}{x}\,dx$ $\qquad$ IF $x=T$, $u = \ln T$; $\quad x = \frac{1}{e}$, $u = \ln\frac{1}{e} = -1$

$$= \lim_{T\to 0^+} 24\int_T^{1/e}\frac{1}{(\ln x)^4}\cdot\frac{1}{x}\,dx = \lim_{T\to 0^+} 24\int_{\ln T}^{-1}\frac{1}{u^4}\,du = \lim_{T\to 0^+} 24\int_{\ln T}^{-1} u^{-4}\,du$$

$$= \lim_{T\to 0^+} 24\left[-\frac{1}{3}u^{-3}\right]_{\ln T}^{-1} = \lim_{T\to 0^+} -8\left(\frac{1}{(-1)^3} - \frac{1}{(\ln T)^3}\right) = -8(-1-0) = \boxed{8}$$

(AS $T\to 0^+$, $\ln T \to -\infty$ SO $\frac{1}{(\ln T)^3} \to 0$)

B) $$\int_{3/2}^{\infty}\frac{15}{2x^2+3x}\,dx = \lim_{T\to\infty}\int_{3/2}^{T}\frac{15}{x(2x+3)}\,dx$$

$$\frac{15}{x(2x+3)} = \frac{A}{x} + \frac{B}{2x+3}$$

$$15 = A(2x+3) + Bx$$

$x=0$: $15 = 3A$ $\quad \underline{A=5}$

$\underline{x=-3/2}$: $15 = -\frac{3}{2}B$ $\quad \underline{B=-10}$

$$= \lim_{T\to\infty}\int_{3/2}^{T}\left(\frac{5}{x} - \frac{10}{2x+3}\right)dx$$

$$= \lim_{T\to\infty} 5\int_{3/2}^{T}\left(\frac{1}{x} - \frac{2}{2x+3}\right)dx = \lim_{T\to\infty} 5\Big[\ln x - \ln(2x+3)\Big]_{3/2}^{T}$$

$$= \lim_{T\to\infty} 5\left[\ln\left(\frac{x}{2x+3}\right)\right]_{3/2}^{T} = \lim_{T\to\infty} 5\left(\ln\left(\frac{T}{2T+3}\right) - \ln\left(\frac{3/2}{6}\right)\right)$$

(AS $T\to\infty$, $\frac{T}{2T+3} \to \frac{1}{2}$)

$$= 5\left(\ln\frac{1}{2} - \ln\frac{1}{4}\right) = 5\ln\frac{1/2}{1/4} = \boxed{5\ln 2}$$

(5) $\int_0^m \frac{1}{6} e^{-t/6}\,dt = \frac{1}{2}$ GIVES $\frac{1}{6}\left[-6e^{-t/6}\right]_0^m = \frac{1}{2}$, $-\left[e^{-t/6}\right]_0^m = \frac{1}{2}$, $-\left(e^{-m/6}-1\right) = \frac{1}{2}$,

$e^{-m/6} - 1 = -\frac{1}{2}$, $e^{-m/6} = \frac{1}{2}$, $-\frac{m}{6} = \ln\frac{1}{2}$, $m = \boxed{-6\ln\frac{1}{2}\text{ MIN.}} = \boxed{6\ln 2\text{ MIN}}$

(6) $\int x^3(\ln x)^2\,dx \longleftarrow$ LET $u = (\ln x)^2$, $dv = x^3\,dx$

$du = 2\ln x\cdot\frac{1}{x}\,dx$, $v = x^4/4$

$= \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\int x^3\ln x\,dx \longleftarrow$ LET $u = \ln x$, $dv = x^3\,dx$

$du = \frac{1}{x}\,dx$, $v = \frac{x^4}{4}$

$= \frac{x^4}{4}(\ln x)^2 - \frac{1}{2}\left[\frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx\right] = \boxed{\frac{x^4}{4}(\ln x)^2 - \frac{x^4}{8}\ln x + \frac{x^4}{32} + C}$

OR LET $t = \ln x$, $x = e^t$, $dx = e^t\,dt$ TO GET

$\int x^3(\ln x)^2\,dx = \int (e^t)^3\cdot t^2\cdot e^t\,dt = \int t^2 e^{4t}\,dt \longleftarrow$

u		dv
t^2	$+$	$e^{4t}dt$
$2t$	$-$	$\frac{1}{4}e^{4t}$
2	$+$	$\frac{1}{16}e^{4t}$
0		$\frac{1}{64}e^{4t}$

$= \frac{1}{4}t^2e^{4t} - \frac{1}{8}te^{4t} + \frac{1}{32}e^{4t} + C$

$= \boxed{\frac{1}{4}(\ln x)^2\cdot x^4 - \frac{1}{8}(\ln x)\cdot x^4 + \frac{1}{32}x^4 + C}$ (SINCE $e^{4t} = (e^t)^4 = x^4$)

(7) $P(t < 1.5) = \int_0^{1.5} 4te^{-2t}\,dt$ LET $u = 4t$, $dv = e^{-2t}\,dt$

$du = 4\,dt$, $v = -\frac{1}{2}e^{-2t}$

$= \left[-2te^{-2t} - (-2)\int e^{-2t}\,dt\right]_0^{1.5} = \left[-2te^{-2t} + 2\left(-\frac{1}{2}e^{-2t}\right)\right]_0^{1.5}$

$= \left[-2te^{-2t} - e^{-2t}\right]_0^{1.5} = (-3e^{-3} - e^{-3}) - (0-1) = \boxed{1 - 4e^{-3}}$

$= \boxed{1 - \frac{4}{e^3}}$

OR

u		dv
$4t$	$+$	$e^{-2t}dt$
4	$-$	$-\frac{1}{2}e^{-2t}$
0		$\frac{1}{4}e^{-2t}$

(8) $\int x^3\sec x^2\tan x^2\,dx \longleftarrow$ LET $t = x^2$, $dt = 2x\,dx$

$= \frac{1}{2}\int x^2\sec x^2\tan x^2\cdot 2x\,dx$

$= \frac{1}{2}\int t\sec t\tan t\,dt \longleftarrow$ LET $u = t$, $dv = \sec t\tan t\,dt$

$du = dt$, $v = \sec t$

$= \frac{1}{2}\left[t\sec t - \int \sec t\,dt\right]$

$= \frac{1}{2}\left[t\sec t - \ln|\sec t + \tan t|\right] + C$

$= \boxed{\frac{1}{2}\left[x^2\sec x^2 - \ln|\sec x^2 + \tan x^2|\right] + C}$

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(9) $$V = \pi \int_0^{\pi/4} (4\sin x + 5\sec x)^2\, dx = \pi \int_0^{\pi/4} (16\sin^2 x + 40 \sin x \sec x + 25\sec^2 x)\, dx$$

$\sin x \sec x = \sin x \cdot \frac{1}{\cos x} = \tan x$

$$= \pi \int_0^{\pi/4} \left(16 \cdot \tfrac{1}{2}(1-\cos 2x) + 40\tan x + 25\sec^2 x\right) dx$$

$$= \pi \left[8\left(x - \tfrac{1}{2}\sin 2x\right) + 40\left(-\ln|\cos x|\right) + 25\tan x \right]_0^{\pi/4}$$

$$= \pi \left(8\left(\tfrac{\pi}{4} - \tfrac{1}{2}\cdot 1\right) - 40 \ln \tfrac{1}{\sqrt{2}} + 25\cdot 1 - 0 \right)$$

$$= \pi \left(2\pi - 4 - 40\left(-\tfrac{1}{2}\ln 2\right) + 25 \right)$$

$$= \boxed{\pi(2\pi + 20\ln 2 + 21)} = \boxed{2\pi^2 + 20\pi \ln 2 + 21\pi}$$

(10) $$\int \frac{1}{\sqrt{x+5}\,(x+4)^2}\, dx$$ Let $u = \sqrt{x+5}$, so $x = u^2 - 5$, $dx = 2u\, du$

$$= \int \frac{1}{\cancel{u}(u^2-1)^2} \cdot 2\cancel{u}\, du = \int \frac{2}{(u^2-1)^2}\, du$$

$$\frac{2}{(u^2-1)^2} = \frac{2}{(u-1)^2(u+1)^2} = \frac{A}{u-1} + \frac{B}{(u-1)^2} + \frac{C}{u+1} + \frac{D}{(u+1)^2}$$

$$2 = A(u-1)(u+1)^2 + B(u+1)^2 + C(u-1)^2(u+1) + D(u-1)^2$$

$u=1$: $2 = 4B$ $\quad B = 1/2$

$u=-1$: $2 = 4D$ $\quad D = 1/2$

u^3 coeff.: $0 = A + C$ so $A = -C$

$u=0$: $2 = -A + \frac{1}{2} + C + \frac{1}{2} = 2C + 1$ so $2C = 1$, $C = \frac{1}{2}$, $A = -\frac{1}{2}$

$$\int \left(\frac{-1/2}{u-1} + \frac{1/2}{(u-1)^2} + \frac{1/2}{u+1} + \frac{1/2}{(u+1)^2} \right) du$$

$$= \frac{1}{2} \int \left(\frac{-1}{u-1} + \frac{1}{(u-1)^2} + \frac{1}{u+1} + \frac{1}{(u+1)^2} \right) du$$

$$= \frac{1}{2} \left[-\ln|u-1| - (u-1)^{-1} + \ln|u+1| - (u+1)^{-1} \right] + C$$

$$= \boxed{\frac{1}{2} \left[-\ln\left|\sqrt{x+5} - 1\right| - \left(\sqrt{x+5} - 1\right)^{-1} + \ln\left(\sqrt{x+5} + 1\right) - \left(\sqrt{x+5} + 1\right)^{-1} \right] + C}$$

$$= \boxed{\frac{1}{2} \ln \left| \frac{\sqrt{x+5} + 1}{\sqrt{x+5} - 1} \right| - \frac{\sqrt{x+5}}{x+4} + C}$$