ANSWERS - TEST 2

(1) $\int x^2 \sin 3x\,dx = \boxed{-\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C} \longleftarrow$

u		dv
x^2	$+$	$\sin 3x\,dx$
$2x$	$-$	$-\frac{1}{3}\cos 3x$
2	$+$	$-\frac{1}{9}\sin 3x$
0		$\frac{1}{27}\cos 3x$

OR Let $u = x^2$, $dv = \sin 3x\,dx$

$du = 2x\,dx$, $v = -\frac{1}{3}\cos 3x$

$\int x^2\sin 3x\,dx = -\frac{1}{3}x^2\cos 3x - \left(-\frac{2}{3}\right)\int x\cos 3x\,dx$ Let $u = x$, $dv = \cos 3x\,dx$

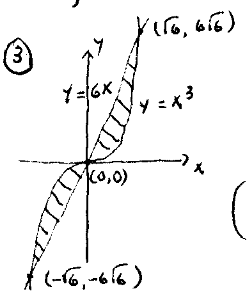
$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx\right]$ $du = dx$, $v = \frac{1}{3}\sin 3x$

$= \boxed{-\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x - \frac{2}{9}\left(-\frac{1}{3}\cos 3x\right) + C} = \boxed{-\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C}$

(2) $\int \frac{32x}{(4x+1)^3}\,dx$ Let $u = 4x+1$, so $x = \frac{1}{4}(u-1)$ and $dx = \frac{1}{4}du$

$= \int \frac{32\cdot\frac{1}{4}(u-1)}{u^3}\cdot\frac{1}{4}\,du = 2\int\frac{u-1}{u^3}\,du = 2\int (u-1)u^{-3}\,du = 2\int (u^{-2} - u^{-3})\,du = 2\left[-u^{-1} + \frac{1}{2}u^{-2}\right] + C$

$= \boxed{-2(4x+1)^{-1} + (4x+1)^{-2} + C}$

(3) 

$x^3 = 6x$, $x^3 - 6x = 0$, $x(x^2-6) = 0$, $x = 0$ or $x^2 = 6$, $x = \pm\sqrt{6}$

$A = 2\int_0^{\sqrt{6}} (6x - x^3)\,dx = 2\left[3x^2 - \frac{x^4}{4}\right]_0^{\sqrt{6}} = 2\left(3\cdot 6 - \frac{36}{4}\right) = 2(18-9) = 2(9) = \boxed{18}$

(OR use $A = \int_{-\sqrt{6}}^{0} (x^3 - 6x)\,dx + \int_0^{\sqrt{6}} (6x - x^3)\,dx$)

(4) A) $\int_0^4 \frac{60t}{(t^2+4)^2}\,dt$ Let $u = t^2+4$, $du = 2t\,dt$ if $t = 0$, $u = 4$; $t = 4$, $u = 20$

$= \frac{60}{2}\int_0^4 \frac{1}{(t^2+4)^2}\cdot 2t\,dt = 30\int_4^{20}\frac{1}{u^2}\,du = 30\left[-\frac{1}{u}\right]_4^{20} = 30\left(-\frac{1}{20} - \left(-\frac{1}{4}\right)\right) = 30\left(\frac{4}{20}\right) = 30\left(\frac{1}{5}\right) = \boxed{6}$

B) $\int_2^3 \frac{27x}{\sqrt{3x-5}}\,dx$ Let $u = \sqrt{3x-5}$, so $x = \frac{1}{3}(u^2+5)$ and $dx = \frac{1}{3}\cdot 2u\,du$ if $x = 2$, $u = 1$; $x = 3$, $u = 2$

$= \int_1^2 \frac{27\cdot\frac{1}{3}(u^2+5)}{u}\cdot\frac{2}{3}u\,du = 6\int_1^2 (u^2+5)\,du = 6\left[\frac{1}{3}u^3 + 5u\right]_1^2 = \left[2u^3 + 30u\right]_1^2 = (16+60) - (2+30) = \boxed{44}$

OR Let $u = 3x-5$, $x = \frac{1}{3}(u+5)$ and $dx = \frac{1}{3}du$ if $x = 2$, $u = 1$ and if $x = 3$, $u = 4$

$\int_2^3 \frac{27x}{\sqrt{3x-5}}\,dx = \int_1^4 \frac{27\cdot\frac{1}{3}(u+5)}{\sqrt{u}}\cdot\frac{1}{3}\,du = 3\int_1^4 \frac{u+5}{\sqrt{u}}\,du = 3\int_1^4 (u+5)u^{-1/2}\,du = 3\int_1^4 (u^{1/2} + 5u^{-1/2})\,du$

$= 3\left[\frac{2}{3}u^{3/2} + 10u^{1/2}\right]_1^4 = \left[2u^{3/2} + 30u^{1/2}\right]_1^4 = (2\cdot 8 + 30\cdot 2) - (2+30) = 76 - 32 = \boxed{44}$

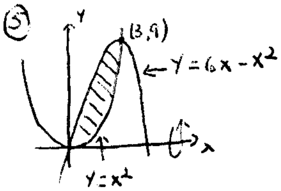
C) $\int_0^{\ln 3} \frac{8}{1+e^{-2x}}\,dx = \int_0^{\ln 3}\frac{8}{1+e^{-2x}}\cdot\frac{e^{2x}}{e^{2x}}\,dx = \int_0^{\ln 3}\frac{8e^{2x}}{e^{2x}+1}\,dx$ Let $u = e^{2x}+1$, $du = 2e^{2x}\,dx$

$= \frac{8}{2}\int_0^{\ln 3}\frac{2e^{2x}}{e^{2x}+1}\,dx = 4\left[\ln(e^{2x}+1)\right]_0^{\ln 3} = 4\left(\ln(e^{2\ln 3}+1) - \ln(1+1)\right) = 4(\ln 10 - \ln 2) = \boxed{4\ln 5}$

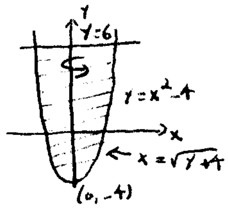
OR change limits to get if $x = 0$, $u = e^0 + 1 = 2$ and if $x = \ln 3$, $u = e^{2\ln 3} + 1 = (e^{\ln 3})^2 + 1 = 3^2 + 1 = 10$

so $\int_0^{\ln 3}\frac{8e^{2x}}{e^{2x}+1}\,dx = \frac{8}{2}\int_0^{\ln 3}\frac{1}{e^{2x}+1}\cdot 2e^{2x}\,dx = 4\int_2^{10}\frac{1}{u}\,du = 4\left[\ln u\right]_2^{10}$

$= 4(\ln 10 - \ln 2) = \boxed{4\ln 5}$

⑤


$$V=\int_0^3 \pi\left((6x-x^2)^2-(x^2)^2\right)dx=\pi\int_0^3(36x^2-12x^3+x^4-x^4)\,dx$$
$$=\pi\left[12x^3-3x^4\right]_0^3=\pi(12\cdot3^3-3\cdot3^4)=\pi(4\cdot3^4-3\cdot3^4)=\pi\cdot3^4=\boxed{81\pi}$$

⑥


$y=x^2-4,\quad x^2=y+4,\quad x=\sqrt{y+4}$ (ON THE RIGHT HALF)

$$V=\int_{-4}^{6}\pi(f(y))^2dy=\int_{-4}^{6}\pi\left(\sqrt{y+4}\right)^2dy=\pi\int_{-4}^{6}(y+4)\,dy=\pi\left[\frac{y^2}{2}+4y\right]_{-4}^{6}$$
$$=\pi\left(18+24-(8-16)\right)=\boxed{50\pi}$$

OR LET $u=y+4,\ du=dy$ TO GET
$$\pi\int_0^{10}u\,du=\pi\left[\frac{u^2}{2}\right]_0^{10}=\pi(50)=\boxed{50\pi}$$

⑦ $\int t\ln(t+3)\,dt$ LET $u=\ln(t+3),\ dv=t\,dt$ TO GET
$du=\frac{1}{t+3}dt,\quad v=\frac{t^2}{2}$

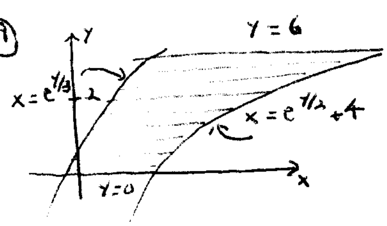
$$\begin{array}{r} t-3 \\ t+3\,\overline{)\,t^2\qquad} \\ \underline{t^2+3t} \\ -3t \\ \underline{-3t-9} \\ 9 \end{array}$$

$$\frac{t^2}{2}\ln(t+3)-\frac12\int\frac{t^2}{t+3}dt=\frac{t^2}{2}\ln(t+3)-\frac12\int\left(t-3+\frac{9}{t+3}\right)dt$$
$$=\boxed{\frac{t^2}{2}\ln(t+3)-\frac12\left[\frac{t^2}{2}-3t+9\ln(t+3)\right]+C}$$
$$=\boxed{\frac{t^2}{2}\ln(t+3)-\frac{t^2}{4}+\frac32t-\frac92\ln(t+3)+C}$$

OR LET $x=t+3,\ t=x-3,\ dt=dx$ TO GET
$\int(x-3)\ln x\,dx$ LET $u=\ln x,\ dv=(x-3)\,dx$
$du=\frac1x dx,\quad v=\frac{x^2}{2}-3x$

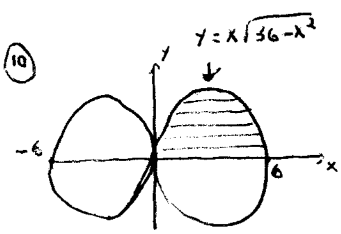
$$=\left(\frac{x^2}{2}-3x\right)\ln x-\int\left(\frac{x}{2}-3\right)dx$$
$$=\left(\frac{x^2}{2}-3x\right)\ln x-\frac{x^2}{4}+3x+C=\boxed{\left(\frac{(t+3)^2}{2}-3(t+3)\right)\ln(t+3)-\frac{(t+3)^2}{4}+3(t+3)+C}$$

⑧ $$\int e^x\cot(e^x)\,dx=\int\cot(e^x)\cdot e^x\,dx=\int\cot u\,du=\ln|\sin u|+C=\boxed{\ln|\sin(e^x)|+C}$$
$u=e^x,\ du=e^x\,dx$

⑨


$y=3\ln(x+2),\quad \ln(x+2)=\frac{y}{3},\quad x+2=e^{y/3},\quad x=e^{y/3}-2$
$y=2\ln(x-4),\quad \ln(x-4)=\frac{y}{2},\quad x-4=e^{y/2},\quad x=e^{y/2}+4$

$$A=\int_0^6\left((e^{y/2}+4)-(e^{y/3}-2)\right)dy=\int_0^6\left(e^{y/2}-e^{y/3}+6\right)dy$$
$$=\left[2e^{y/2}-3e^{y/3}+6y\right]_0^6=(2e^3-3e^2+36)-(2-3)=\boxed{2e^3-3e^2+37}$$

⑩


$y^2=x^2(36-x^2)$ SO $y=\pm x\sqrt{36-x^2}$ AND $y=0$ FOR $x=0$ OR $x^2=36,\ x=\pm6$

$$A=4\int_0^6x\sqrt{36-x^2}\,dx$$ LET $u=36-x^2,\ du=-2x\,dx$ IF $x=0,\ u=36$; $x=6,\ u=0$

$$=4\left(-\frac12\right)\int_0^6\sqrt{36-x^2}\,(-2)x\,dx=-2\int_{36}^{0}\sqrt{u}\,du=2\int_0^{36}u^{1/2}\,du$$
$$=2\left[\frac23u^{3/2}\right]_0^{36}=\frac43\left(36^{3/2}\right)=\frac43\cdot6^3=\frac43\cdot6\cdot6^2=8\cdot36=\boxed{288}$$