ANSWERS – TEST 3

① $\int_2^{42} \frac{1}{x+3}\,dx$, $n=4$

7 17 27 37

2 12 22 32 42

$\Delta x = \frac{42-2}{4} = 10$

A) MIDPOINT RULE

$$\int_2^{42} \frac{1}{x+3}\,dx \approx \boxed{\frac{1}{10}\cdot 10 + \frac{1}{20}\cdot 10 + \frac{1}{30}\cdot 10 + \frac{1}{40}\cdot 10} = \boxed{\left[\frac{1}{10}+\frac{1}{20}+\frac{1}{30}+\frac{1}{40}\right]\cdot 10}$$

B) SIMPSON'S RULE

$$\int_2^{42} \frac{1}{x+3}\,dx \approx \boxed{\frac{10}{3}\left[\frac{1}{5} + 4\cdot\frac{1}{15} + 2\cdot\frac{1}{25} + 4\cdot\frac{1}{35} + \frac{1}{45}\right]}$$

② $f(x) = \frac{K}{x^2}$, $[3,12]$

A) $\int_3^{12} \frac{K}{x^2}\,dx = 1$, $K\left[-\frac{1}{x}\right]_3^{12} = 1$, $K\left(-\frac{1}{12} - \left(-\frac{1}{3}\right)\right) = 1$, $K\cdot\frac{1}{4} = 1$, $\boxed{K=4}$

B) $P(x \le 8) = \int_3^8 \frac{4}{x^2}\,dx = 4\left[-\frac{1}{x}\right]_3^8 = 4\left(-\frac{1}{8} - \left(-\frac{1}{3}\right)\right) = -\frac{1}{2} + \frac{4}{3} = \boxed{\frac{5}{6}}$

③ $\int \frac{x^2+20x-36}{x(x-3)^2}\,dx$

$$\frac{x^2+20x-36}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$x^2+20x-36 = A(x-3)^2 + Bx(x-3) + Cx$$

$x=0$: $-36 = 9A$ $\quad A = -4$

$x=3$: $33 = 3C$ $\quad C = 11$

x^2 COEFF.: $1 = A + B$ $\quad B = 5$

$\int \frac{11}{(x-3)^2}\,dx = \int \frac{11}{u^2}\,du$

$= 11\left(-\frac{1}{u}\right) + C$

FOR $u = x-3$, $du = dx$

$$\int\left(-\frac{4}{x} + \frac{5}{x-3} + \frac{11}{(x-3)^2}\right)dx = \boxed{-4\ln|x| + 5\ln|x-3| - 11(x-3)^{-1} + C}$$

$$= \boxed{-4\ln|x| + 5\ln|x-3| - \frac{11}{x-3} + C}$$

④ $f(T) = \frac{1}{8}e^{-T/8}$, $[0,\infty)$, so

$$P(T<16) = \int_0^{16} \frac{1}{8}e^{-T/8}\,dT = \frac{1}{8}\left[-8e^{-T/8}\right]_0^{16} = -\left(e^{-2}-1\right) = \boxed{1-e^{-2}} = \boxed{1-\frac{1}{e^2}}$$

⑤ $\int_1^{e^2} \frac{\ln x}{x^2}\,dx$

LET $u = \ln x$, $dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{x}\,dx$, $v = -\frac{1}{x}$

$\ln e^2 = 2$ AND $\ln 1 = 0$

$$= \left[(\ln x)\left(-\frac{1}{x}\right) - \int -\frac{1}{x^2}\,dx\right]_1^{e^2} = \left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^{e^2} = \left(-\frac{2}{e^2} - \frac{1}{e^2}\right) - (0-1) = \boxed{1 - \frac{3}{e^2}}$$

⑥ $\int \frac{2x^3-14}{x^2-x-6}\,dx$

$$\begin{array}{r} 2x + 2 \\ x^2-x-6\ \overline{)\ 2x^3 \qquad\qquad -14} \\ \underline{2x^3 - 2x^2 - 12x} \\ 2x^2 + 12x - 14 \\ \underline{2x^2 - 2x - 12} \\ 14x - 2 \end{array}$$

$$= \int\left(2x+2 + \frac{14x-2}{x^2-x-6}\right)dx$$

$$= x^2 + 2x + \int \frac{14x-2}{(x-3)(x+2)}\,dx$$

$$\frac{14x-2}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$14x-2 = A(x+2) + B(x-3)$$

$x=3$: $40 = 5A$ $\quad A = 8$

$x=-2$: $-30 = -5B$ $\quad B = 6$

$$= x^2 + 2x + \int\left(\frac{8}{x-3} + \frac{6}{x+2}\right)dx$$

$$= \boxed{x^2 + 2x + 8\ln|x-3| + 6\ln|x+2| + C}$$

⑦ $\int \sin^8\theta \cos^3\theta \, d\theta = \int \sin^8\theta \cos^2\theta \cos\theta \, d\theta = \int \sin^8\theta \, (1-\sin^2\theta) \cos\theta \, d\theta$ LET $u = \sin\theta$, $du = \cos\theta \, d\theta$

$= \int u^8 (1-u^2) \, du = \int (u^8 - u^{10}) \, du = \frac{1}{9}u^9 - \frac{1}{11}u^{11} + C$

$= \boxed{\frac{1}{9}\sin^9\theta - \frac{1}{11}\sin^{11}\theta + C}$

⑧ $\int_{4/3}^{\infty} \frac{25}{(3x+1)(x+2)} \, dx = \lim_{T\to\infty} \int_{4/3}^{T} \frac{25}{(3x+1)(x+2)} \, dx$

$\frac{25}{(3x+1)(x+2)} = \frac{A}{3x+1} + \frac{B}{x+2}$

$25 = A(x+2) + B(3x+1)$

$x=-2$: $25 = -5B$ $\quad B = -5$

$x = 0$: $25 = 2A + B = 2A - 5$ $\quad A = 15$

$= \lim_{T\to\infty} \int_{4/3}^{T} \left(\frac{15}{3x+1} - \frac{5}{x+2} \right) dx$

$= \lim_{T\to\infty} 5 \int_{4/3}^{T} \left(\frac{3}{3x+1} - \frac{1}{x+2} \right) dx$

$= \lim_{T\to\infty} 5 \left[\ln(3x+1) - \ln(x+2) \right]_{4/3}^{T} = \lim_{T\to\infty} 5 \left[\ln\left(\frac{3x+1}{x+2}\right) \right]_{4/3}^{T}$

$= \lim_{T\to\infty} 5 \left(\ln\left(\frac{3T+1}{T+2}\right) - \ln\frac{5}{10/3} \right) = 5\left(\ln 3 - \ln\frac{3}{2}\right) = 5\ln\frac{3}{3/2} = \boxed{5\ln 2}$

(SINCE $\frac{3T+1}{T+2} \to 3$ AS $T \to \infty$)

⑨ $\int x^5 \sec^2(x^3) \, dx$ LET $T = x^3$, $dT = 3x^2 \, dx$

$= \frac{1}{3} \int x^3 \sec^2(x^3) \cdot 3 x^2 \, dx$

$= \frac{1}{3} \int T \sec^2 T \, dT$ LET $u = T$, $dV = \sec^2 T \, dT$; $du = dT$, $V = \tan T$

$= \frac{1}{3}\left[T\tan T - (-\ln|\cos T|) \right] + C$ $\leftarrow$ OR use $\frac{1}{3}\left[T\tan T - \ln|\sec T| \right] + C$

$= \boxed{\frac{1}{3}\left[x^3 \tan x^3 + \ln|\cos x^3| \right] + C}$ $\quad = \boxed{\frac{1}{3}\left[x^3 \tan x^3 - \ln|\sec x^3| \right] + C}$

⑩ $\mu = E(T) = \int_2^6 T \cdot \frac{1}{2\sqrt{2T-3}} \, dT$ LET $u = \sqrt{2T-3}$, SO $T = \frac{1}{2}(u^2+3)$, $dT = \frac{1}{2} \cdot 2u\,du = u\,du$

IF $T=2$, $u=1$; $T=6$, $u=3$

$= \int_1^3 \frac{1}{2}(u^2+3) \cdot \frac{1}{2\cancel{u}} \cdot \cancel{u}\,du = \frac{1}{4}\int_1^3 (u^2+3)\,du$

$= \frac{1}{4}\left[\frac{u^3}{3} + 3u \right]_1^3 = \frac{1}{4}\left((9+9) - \left(\frac{1}{3}+3\right) \right) = \frac{1}{4}\left(15 - \frac{1}{3}\right) = \frac{1}{4}\cdot\frac{44}{3} = \boxed{\frac{11}{3} \text{ DAYS}}$

(SEE P. 3 FOR AN ALTERNATE SOLUTION.)

(10) (ALTERNATE SOLUTION)

LET $u = 2T - 3$, $T = \frac{1}{2}(u+3)$, $dT = \frac{1}{2}\,du$

IF $T = 2,\ u = 1$
$T = 6,\ u = 9$

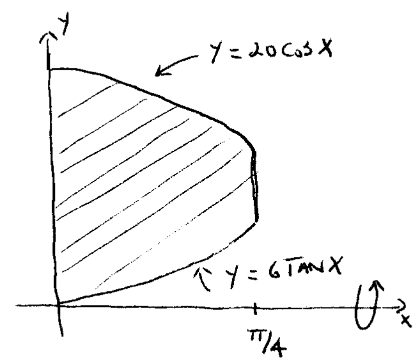
$$\mu = E(T) = \int_2^6 T \cdot \frac{1}{2\sqrt{2T-3}}\,dT$$

$$= \int_1^9 \frac{1}{2}(u+3)\cdot\frac{1}{2\sqrt{u}}\cdot\frac{1}{2}\,du = \frac{1}{8}\int_1^9 \frac{u+3}{\sqrt{u}}\,du = \frac{1}{8}\int_1^9 \left(\frac{u}{\sqrt{u}} + \frac{3}{\sqrt{u}}\right)du$$

$$= \frac{1}{8}\int_1^9 \left(u^{1/2} + 3u^{-1/2}\right)du = \frac{1}{8}\left[\frac{2}{3}u^{3/2} + 6u^{1/2}\right]_1^9 = \frac{1}{8}\left(\left(\frac{2}{3}\cdot 27 + 6\cdot 3\right) - \left(\frac{2}{3} + 6\right)\right)$$

$$= \frac{1}{8}\left((18+18) - \left(\frac{2}{3}+6\right)\right) = \frac{1}{8}\left(30 - \frac{2}{3}\right) = \frac{1}{8}\cdot\frac{88}{3} = \boxed{\frac{11}{3}\text{ DAYS}}$$

(11)



$$V = \pi\int_0^{\pi/4}\left((20\cos x)^2 - (6\tan x)^2\right)dx$$

$$= \pi\int_0^{\pi/4}\left(400\cos^2 x - 36\tan^2 x\right)dx$$

$$= \pi\int_0^{\pi/4}\left(400\cdot\frac{1}{2}(1+\cos 2x) - 36(\sec^2 x - 1)\right)dx$$

$$= \pi\int_0^{\pi/4}\left(200(1+\cos 2x) - 36(\sec^2 x - 1)\right)dx$$

$$= \pi\left[200\left(x + \frac{1}{2}\sin 2x\right) - 36(\tan x - x)\right]_0^{\pi/4}$$

$$= \pi\left[200x + 100\sin 2x - 36\tan x + 36x\right]_0^{\pi/4}$$

$$= \pi\left(50\pi + 100\sin\frac{\pi}{2} - 36\tan\frac{\pi}{4} + 9\pi - 0\right)$$

$$= \pi\left(50\pi + 100\cdot 1 - 36\cdot 1 + 9\pi\right)$$

$$= \boxed{\pi(59\pi + 64)} = \boxed{59\pi^2 + 64\pi}$$