For each of the following functions,

A) Find all of the critical points.

B) Classify each critical point as a rel. max., rel. min., or saddle point.

1. \( f(x,y) = x^3 - 6xy + y^3 \)
2. \( f(x,y) = x^3 + 4xy + 2y^4 \)
3. \( f(x,y) = 6x^2 + 6y^2 - 3x^2y - y^3 \)
4. \( f(x,y) = x^3 - 2y^3 - 27x + 24y \)
5. \( f(x,y) = 4x^2e^y - 2x^4 - e^{2y} \)
6. \( f(x,y) = e^{3y} + x^3 - 3xey \)

For the following functions,

i) Determine the relative extrema for \( f \).

ii) Identify a property that \( f \) has that a continuous function of one variable cannot have.

1. \( f(x,y) = 4x^2e^y - 2x^4 - e^{2y} \)
2. \( f(x,y) = e^{3y} + x^3 - 3xey \)

(IN EACH OF THE FOLLOWING, VERIFY THAT YOUR ANSWER IS A REL. MINIMUM.)

Find the point on the plane \( 3x + 2y - z = -17 \) which is closest to the point \( (5, 2, 8) \).

Find the dimensions of the rectangular box with an open top and a volume of 32 cm\(^3\) which has minimal surface area.

A closed rectangular box with a volume of 60 ft\(^3\) is to be made of material which costs \$2/ft\(^2\) for the bottom, \$1/ft\(^2\) for the top, and only \$0.20/ft\(^2\) for the sides. Find the dimensions of the least expensive such box.

Let \( l_1 \) and \( l_2 \) be the lines in 3-space given by the following sets of equations:

\( l_1 : \begin{align*} x &= 2t, \quad y = 1 + t, \quad z = 2 - t \end{align*} \)
\( l_2 : \begin{align*} x &= 4 + 5s, \quad y = -1 - s, \quad z = 1 - 5s \end{align*} \)

Find the points on \( l_1 \) and \( l_2 \) which are closest to each other.