I) To find the extrema of \( f(x,y) \) on the curve \( g(x,y) = k \),

Solve the system:

\[
\begin{align*}
\frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} \\
\frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} \\
g(x,y) &= k
\end{align*}
\]

Geometric justification:

At a point on the curve \( g(x,y) = k \) where \( f \) has a max. or a min.,

the level curve of \( f \) is tangent to the constraint curve \( g(x,y) = k \).

This means that the perpendicular direction \( \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle \) to the level curve of \( f \)

is a multiple of the perpendicular direction \( \langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \rangle \) to the constraint curve,

so \( \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle = \lambda \langle \frac{\partial g}{\partial x}, \frac{\partial g}{\partial y} \rangle \) for some number \( \lambda \) and therefore \( \frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x} \)

and \( \frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y} \).

II) To find the extrema of \( f(x,y,z) \) on the surface \( g(x,y,z) = k \),

Solve the system:

\[
\begin{align*}
\frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} \\
\frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} \\
\frac{\partial f}{\partial z} &= \lambda \frac{\partial g}{\partial z} \\
g(x,y,z) &= k
\end{align*}
\]

Basic strategies:

A) Solve for \( \lambda \) in each equation and set the resulting expressions equal,

a) solve for two of the variables \( x, y, z \) in terms of the 3rd variable,

b) substitute into the constraint equation to solve for the 3rd variable,

and then find the corresponding values of the other two variables.

OR

B) Solve for each variable in terms of \( \lambda \),

a) substitute into the constraint equation to solve for \( \lambda \),

b) find the corresponding values of \( x, y, \) and \( z \),