MATH 17A ANSWERS - WHITE SHEET (SEC. 5.1 − 5.2)

(11) $h(x) = \sqrt{3}\sin x + \cos x$, x in $(0, 2\pi)$

$h'(x) = \sqrt{3}\cos x - \sin x = 0$ IF $\sqrt{3}\cos x = \sin x$, $\sqrt{3} = \dfrac{\sin x}{\cos x}$, $\tan x = \sqrt{3}$

CRITICAL NUMBERS: $\boxed{x = \frac{\pi}{3}}$ AND $\boxed{x = \frac{4\pi}{3}} \leftarrow \pi + \frac{\pi}{3}$

0		$\pi/3$		$4\pi/3$		2π
h'	$+$		$-$		$+$	

1) $h'(0) = \sqrt{3}\cdot 1 - 0 = \sqrt{3}$
2) $h'(\frac{\pi}{2}) = \sqrt{3}\cdot 0 - 1 = -1$
3) $h'(2\pi) = \sqrt{3}\cdot 1 - 0 = \sqrt{3}$

$h(\frac{\pi}{3}) = 2$ IS A LOCAL MAX. $\leftarrow h(\frac{\pi}{3}) = \sqrt{3}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2} = 2$

$h(\frac{4\pi}{3}) = -2$ IS A LOCAL MIN. $\leftarrow h(\frac{4\pi}{3}) = \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right) = -2$

(12) $f(x) = \dfrac{x^2 - 3x + 11}{x - 2}$ $f'(x) = \dfrac{(x-2)(2x-3) - (x^2 - 3x + 11)\cdot 1}{(x-2)^2} = \dfrac{x^2 - 4x - 5}{(x-2)^2} = \dfrac{(x-5)(x+1)}{(x-2)^2}$

	-1		2		5	
$+$		$-$	$\times$	$-$		$+$ f'

$f'(6) = \dfrac{1\cdot 7}{16}$

CRITICAL NUMBERS: $\boxed{-1 \text{ AND } 5}$

$f(-1) = -5$ IS A LOCAL MAX.
$f(5) = 7$ IS A LOCAL MIN.

(13) $g(x) = 3x^{8/3} - 12x^{2/3}$

$g'(x) = 8x^{5/3} - 8x^{-1/3} = 8\left(x^{5/3} - \dfrac{1}{x^{1/3}}\right) = 8\left(\dfrac{x^2 - 1}{x^{1/3}}\right) = 8\left(\dfrac{(x-1)(x+1)}{x^{1/3}}\right)$

CRITICAL NUMBERS: $\boxed{-1, 0, 1}$ (SINCE $g'(-1) = 0$, $g'(1) = 0$, AND $g'(0)$ DOES NOT EXIST)

	-1		0		1	
$-$		$+$		$-$		$+$ g'

$g'(2) = \dfrac{8\cdot 1\cdot 3}{2^{1/3}}$

$g(-1) = -9$ IS A LOCAL MIN.
$g(0) = 0$ IS A LOCAL MAX.
$g(1) = -9$ IS A LOCAL MIN.

(14) $h(x) = \sin 2x + 2\cos x$, x in $(0, 2\pi)$

$h'(x) = \cos 2x \cdot 2 - 2\sin x = 2(1 - 2\sin^2 x - \sin x) = -2(2\sin^2 x + \sin x - 1)$

$= -2(2\sin x - 1)(\sin x + 1) = 0$ IF 1) $\sin x = \frac{1}{2}$ OR 2) $\sin x = -1$

1) IF $\sin x = \frac{1}{2}$, $x = \frac{\pi}{6}$ OR $x = \frac{5\pi}{6}$ 2) IF $\sin x = -1$, $x = \frac{3\pi}{2}$

CRITICAL NUMBERS: $\boxed{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

0		$\pi/6$		$5\pi/6$		$3\pi/2$		2π
h'	$+$		$-$		$+$		$+$	

1) $h'(0) = (-2)(-1)(1) = 2$
2) $h'(\frac{\pi}{2}) = (-2)(1)(2) = -4$
3) $h'(\pi) = (-2)(-1)(1) = 2$
4) $h'(2\pi) = (-2)(-1)(1) = 2$

$h(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}$ IS A LOCAL MAX. $\leftarrow h(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} + 2\left(\frac{\sqrt{3}}{2}\right)$

$h(\frac{5\pi}{6}) = -\frac{3\sqrt{3}}{2}$ IS A LOCAL MIN. $\leftarrow h(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2} + 2\left(-\frac{\sqrt{3}}{2}\right)$

MATH 17A ANSWERS – PINK SHEET (SEC. 5.2)

① $f(x) = x^4 - 4x^3$ $\quad f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

f'': + on $(-\infty, 0)$, − on $(0, 2)$, + on $(2, \infty)$; $f''(1) = -12$

$\boxed{\begin{array}{l} \text{CU ON } (-\infty, 0) \text{ AND } (2, \infty) \\ \text{CD ON } (0, 2) \end{array}}$

PTS OF INFLECTION : $\boxed{(0,0) \text{ AND } (2,-16)}$

② $f(x) = x^4 - 2x^3 - 12x^2$ $\quad f'(x) = 4x^3 - 6x^2 - 24x$

$f''(x) = 12x^2 - 12x - 24 = 12(x^2 - x - 2) = 12(x-2)(x+1)$

f'': + on $(-\infty, -1)$, − on $(-1, 2)$, + on $(2, \infty)$; $f''(0) = -24$

$\boxed{\begin{array}{l} \text{CU ON } (-\infty, -1) \text{ AND } (2, \infty) \\ \text{CD ON } (-1, 2) \end{array}}$

POINTS OF INFLECTION: $\boxed{(-1,-9) \text{ AND } (2,-48)}$

③ $f(x) = 10x^3 - 3x^5$ $\quad f'(x) = 30x^2 - 15x^4$

$f''(x) = 60x - 60x^3 = 60x(1-x^2) = 60x(1-x)(1+x)$

f'': + on $(-\infty, -1)$, − on $(-1, 0)$, + on $(0, 1)$, − on $(1, \infty)$; $f''(2) = 120(-1)(3)$

$\boxed{\begin{array}{l} \text{CU ON } (-\infty, -1) \text{ AND } (0, 1) \\ \text{CD ON } (-1, 0) \text{ AND } (1, \infty) \end{array}}$

PTS OF INFLECTION : $\boxed{(-1,-7), (0,0), (1,7)}$

④ $f(x) = \dfrac{80}{x^2+12} = 80(x^2+12)^{-1}$ $\quad f'(x) = -80(x^2+12)^{-2} \cdot 2x = -160\left(\dfrac{x}{(x^2+12)^2}\right)$

$$f''(x) = -160\left(\frac{(x^2+12)^2 \cdot 1 - x(2(x^2+12) \cdot 2x)}{(x^2+12)^4}\right) = -160\left(\frac{\cancel{(x^2+12)}(x^2+12-4x^2)}{(x^2+12)^{\cancel{4}\,3}}\right)$$

$$= \frac{-160(12-3x^2)}{(x^2+12)^3} = \frac{480(x^2-4)}{(x^2+12)^3} = \frac{480(x-2)(x+2)}{(x^2+12)^3}$$

f'': + on $(-\infty, -2)$, − on $(-2, 2)$, + on $(2, \infty)$; $f''(0) = \dfrac{480(-4)}{12^3}$

$\boxed{\begin{array}{l} \text{CU ON } (-\infty, -2) \text{ AND } (2, \infty) \\ \text{CD ON } (-2, 2) \end{array}}$

PTS OF INFLECTION : $\boxed{(-2,5) \text{ AND } (2,5)}$

⑧ A) $f' > 0$, $f'' > 0$ $\qquad$ B) $f' < 0$, $f'' > 0$ $\qquad$ C) $f' > 0$, $f'' < 0$ $\qquad$ D) $f' < 0$, $f'' < 0$

⑨ $f(x) = \dfrac{6x}{x^2-4} = \dfrac{6x}{(x-2)(x+2)}$

HORIZONTAL ASYMP. : $\boxed{y = 0}$

VERTICAL ASYMP. : $\boxed{x = 2 ,\ x = -2}$

(10) $f(x) = \dfrac{4x^2 - x + 7}{9 - x^2} = \dfrac{4x^2 - x + 7}{(3-x)(3+x)}$

VERTICAL ASYMP.: $\boxed{x = 3,\ x = -3}$

HORIZONTAL ASYMP.: $\boxed{y = -4}$

(11) $f(x) = \dfrac{2x^2 + x - 1}{x^2 - 2x - 3} = \dfrac{(2x-1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}} = \dfrac{2x-1}{x-3}$

VERTICAL ASYMP.: $\boxed{x = 3}$

HORIZONTAL ASYMP.: $\boxed{y = 2}$

(12) $f(x) = \dfrac{x^3 + x}{x^2 + 4}$ VERTICAL ASYMP.: NONE (SINCE $x^2 + 4 \neq 0$ FOR ALL x)

$$\begin{array}{r|l} & x \\ \hline x^2+4 & x^3 + x \\ & x^3 + 4x \\ \hline & -3x \end{array}$$

$\leftarrow f(x) = x + \dfrac{-3x}{x^2+4}$

SLANTED ASYMP.: $\boxed{y = x}$

(13) $f(x) = \dfrac{x^3}{x^2 + x - 6} = \dfrac{x^3}{(x+3)(x-2)}$

VERTICAL ASYMP.: $\boxed{x = -3,\ x = 2}$

$$\begin{array}{r|l} & x - 1 \\ \hline x^2+x-6 & x^3 \\ & x^3 + x^2 - 6x \\ \hline & -x^2 + 6x \\ & -x^2 - x + 6 \\ \hline & 7x - 6 \end{array}$$

$\leftarrow f(x) = x - 1 + \dfrac{7x - 6}{x^2 + x - 6}$

SLANTED ASYMP.: $\boxed{y = x - 1}$

(14) $f(x) = \dfrac{10x}{\sqrt{4x^2+1}}$ VERTICAL ASYMP.: NONE (SINCE $4x^2 + 1 \neq 0$ FOR ALL x)

a) $\displaystyle \lim_{x\to\infty} \frac{10x}{\sqrt{4x^2+1}} = \lim_{x\to\infty} \frac{10x}{\sqrt{x^2}\sqrt{4+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{10x}{|x|\sqrt{4+\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{10\cancel{x}}{\cancel{x}\sqrt{4+\frac{1}{x^2}}}$

$\displaystyle = \lim_{x\to\infty} \frac{10}{\sqrt{4+\frac{1}{x^2}}} = \frac{10}{2} = \underline{5}$, so $\boxed{y = 5}$ is a HORIZONTAL ASYMP.

b) $\displaystyle \lim_{x\to-\infty} \frac{10x}{\sqrt{4x^2+1}} = \lim_{x\to-\infty} \frac{10x}{\sqrt{x^2}\sqrt{4+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{10x}{|x|\sqrt{4+\frac{1}{x^2}}} = \lim_{x\to-\infty} \frac{10\cancel{x}}{(-\cancel{x})\sqrt{4+\frac{1}{x^2}}}$

$\displaystyle = \lim_{x\to-\infty} \frac{10}{-\sqrt{4+\frac{1}{x^2}}} = \frac{10}{-2} = \underline{-5}$, so $\boxed{y = -5}$ is ALSO A HORIZONTAL ASYMP.