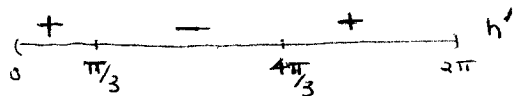


11) $h(x) = \sqrt{3} \sin x + \cos x$, x in $(0, 2\pi)$

$h'(x) = \sqrt{3} \cos x - \sin x = 0$ IF $\sqrt{3} \cos x = \sin x$, $\sqrt{3} = \frac{\sin x}{\cos x}$, $\tan x = \sqrt{3}$

CRITICAL NUMBERS: $x = \frac{\pi}{3}$ AND $x = \frac{4\pi}{3} \leftarrow \pi + \frac{\pi}{3}$



1) $h'(0) = \sqrt{3} \cdot 1 - 0 = \sqrt{3}$

2) $h'(\frac{\pi}{2}) = \sqrt{3} \cdot 0 - 1 = -1$

3) $h'(2\pi) = \sqrt{3} \cdot 1 - 0 = \sqrt{3}$

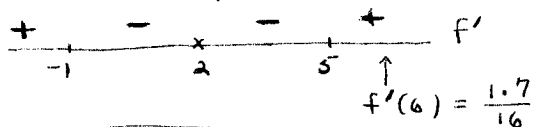
$h(\frac{\pi}{3}) = 2$ IS A LOCAL MAX.
 $h(\frac{4\pi}{3}) = -2$ IS A LOCAL MIN.

$\leftarrow h(\frac{\pi}{3}) = \sqrt{3} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} = 2$

$\leftarrow h(\frac{4\pi}{3}) = \sqrt{3} \cdot (-\frac{\sqrt{3}}{2}) + (-\frac{1}{2}) = -2$

12)

$f(x) = \frac{x^2 - 3x + 11}{x - 2}$ $f'(x) = \frac{(x-2)(2x-3) - (x^2-3x+11) \cdot 1}{(x-2)^2} = \frac{x^2 - 4x - 5}{(x-2)^2} = \frac{(x-5)(x+1)}{(x-2)^2}$



CRITICAL NUMBERS: -1 AND 5

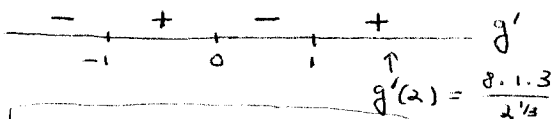
$f(-1) = -5$ IS A LOCAL MAX.
 $f(5) = 7$ IS A LOCAL MIN.

13)

$g(x) = 3x^{8/3} - 12x^{2/3}$

$g'(x) = 8x^{5/3} - 8x^{-1/3} = 8(x^{5/3} - \frac{1}{x^{1/3}}) = 8(\frac{x^2 - 1}{x^{1/3}}) = 8(\frac{(x-1)(x+1)}{x^{1/3}})$

CRITICAL NUMBERS: $-1, 0, 1$ (since $g'(-1) = 0$, $g'(1) = 0$, AND $g'(0)$ DOES NOT EXIST)



$g(-1) = -9$ IS A LOCAL MIN.
 $g(0) = 0$ IS A LOCAL MAX.
 $g(1) = -9$ IS A LOCAL MIN.

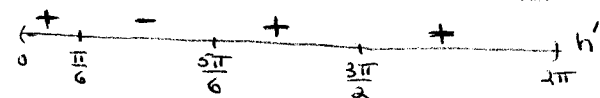
14)

$h(x) = 5 \sin 2x + 2 \cos x$, x in $(0, 2\pi)$

$h'(x) = \cos 2x \cdot 2 - 2 \sin x = 2(1 - 2 \sin^2 x - \sin x) = -2(2 \sin^2 x + \sin x - 1)$
 $= -2(2 \sin x - 1)(\sin x + 1) = 0$ IF 1) $\sin x = \frac{1}{2}$ OR 2) $\sin x = -1$

1) IF $\sin x = \frac{1}{2}$, $x = \frac{\pi}{6}$ OR $x = \frac{5\pi}{6}$ 2) IF $\sin x = -1$, $x = \frac{3\pi}{2}$

CRITICAL NUMBERS: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$



1) $h'(0) = (-2)(-1)(1) = 2$

2) $h'(\frac{\pi}{2}) = (-2)(1)(2) = -4$

3) $h'(\pi) = (-2)(-1)(1) = 2$

4) $h'(2\pi) = (-2)(-1)(1) = 2$

$h(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}$ IS A LOCAL MAX.
 $h(\frac{5\pi}{6}) = -\frac{3\sqrt{3}}{2}$ IS A LOCAL MIN.

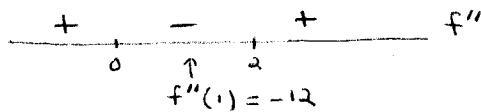
$\leftarrow h(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} + 2(\frac{\sqrt{3}}{2})$

$\leftarrow h(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2} + 2(-\frac{\sqrt{3}}{2})$

① $f(x) = x^4 - 4x^3$ $f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

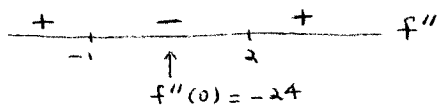
CU ON $(-\infty, 0)$ AND $(2, \infty)$
 CD ON $(0, 2)$



PTS OF INFLECTION: $(0, 0)$ AND $(2, -16)$

② $f(x) = x^4 - 2x^3 - 12x^2$ $f'(x) = 4x^3 - 6x^2 - 24x$

$f''(x) = 12x^2 - 12x - 24 = 12(x^2 - x - 2) = 12(x-2)(x+1)$

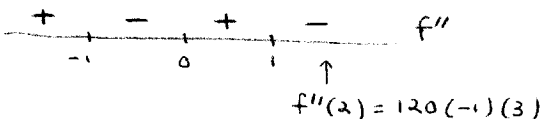


CU ON $(-\infty, -1)$ AND $(2, \infty)$
 CD ON $(-1, 2)$

POINTS OF INFLECTION: $(-1, -9)$ AND $(2, -18)$

③ $f(x) = 10x^3 - 3x^5$ $f'(x) = 30x^2 - 15x^4$

$f''(x) = 60x - 60x^3 = 60x(1-x^2) = 60x(1-x)(1+x)$

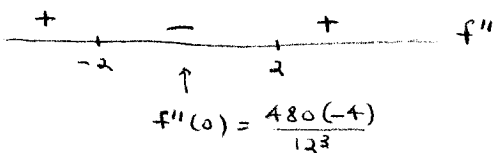


CU ON $(-\infty, -1)$ AND $(0, 1)$
 CD ON $(-1, 0)$ AND $(1, \infty)$

PTS OF INFLECTION: $(-1, -7)$, $(0, 0)$, $(1, 7)$

④ $f(x) = \frac{80}{x^2+12} = 80(x^2+12)^{-1}$ $f'(x) = -80(x^2+12)^{-2} \cdot 2x = -160 \left(\frac{x}{(x^2+12)^2} \right)$

$f''(x) = -160 \left(\frac{(x^2+12)^2 \cdot 1 - x(2(x^2+12) \cdot 2x)}{(x^2+12)^4} \right) = -160 \left(\frac{(x^2+12)(x^2+12-4x^2)}{(x^2+12)^4} \right)$
 $= \frac{-160(12-3x^2)}{(x^2+12)^3} = \frac{480(x^2-4)}{(x^2+12)^3} = \frac{480(x-2)(x+2)}{(x^2+12)^3}$



CU ON $(-\infty, -2)$ AND $(2, \infty)$
 CD ON $(-2, 2)$

PTS OF INFLECTION: $(-2, 5)$ AND $(2, 5)$

- ⑧ a) $f' > 0$ b) $f' < 0$ c) $f' > 0$ d) $f' < 0$
 $f'' > 0$ $f'' > 0$ $f'' < 0$ $f'' < 0$

⑨ $f(x) = \frac{6x}{x^2-4} = \frac{6x}{(x-2)(x+2)}$

HORIZONTAL ASYMP: $y = 0$

VERTICAL ASYMP: $x = 2, x = -2$

(10) $f(x) = \frac{4x^2 - x + 7}{9 - x^2} = \frac{4x^2 - x + 7}{(3-x)(3+x)}$

VERTICAL ASYMP.: $x=3, x=-3$

HORIZONTAL ASYMP.: $y=-1$

(11) $f(x) = \frac{2x^2 + x - 1}{x^2 - 2x - 3} = \frac{(2x-1)(x+1)}{(x-3)(x+1)} = \frac{2x-1}{x-3}$

VERTICAL ASYMP.: $x=3$

HORIZONTAL ASYMP.: $y=2$

(12) $f(x) = \frac{x^3 + x}{x^2 + 4}$ VERTICAL ASYMP.: NONE (since $x^2 + 4 \neq 0$ FOR ALL x)

$x^2 + 4 \overline{) \begin{array}{r} x^3 + x \\ x^3 + 4x \\ \hline -3x \end{array}}$ $\leftarrow f(x) = x + \frac{-3x}{x^2 + 4}$

SLANTED ASYMP.: $y=x$

(13) $f(x) = \frac{x^3}{x^2 + x - 6} = \frac{x^3}{(x+3)(x-2)}$

VERTICAL ASYMP.: $x=-3, x=2$

$x^2 + x - 6 \overline{) \begin{array}{r} x^3 \\ x^3 + x^2 - 6x \\ \hline -x^2 + 6x \\ -x^2 - x + 6 \\ \hline 7x - 6 \end{array}}$ $\leftarrow f(x) = x - 1 + \frac{7x-6}{x^2 + x - 6}$

SLANTED ASYMP.: $y=x-1$

(14) $f(x) = \frac{10x}{\sqrt{4x^2 + 1}}$ VERTICAL ASYMP.: NONE (since $4x^2 + 1 \neq 0$ FOR ALL x)

a) $\lim_{x \rightarrow \infty} \frac{10x}{\sqrt{4x^2 + 1}} = \lim_{x \rightarrow \infty} \frac{10x}{\sqrt{x^2} \sqrt{4 + \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{10x}{|x| \sqrt{4 + \frac{1}{x^2}}} = \lim_{x \rightarrow \infty} \frac{10x}{x \sqrt{4 + \frac{1}{x^2}}}$
 $= \lim_{x \rightarrow \infty} \frac{10}{\sqrt{4 + \frac{1}{x^2}}} = \frac{10}{2} = 5$, so $y=5$ is a HORIZONTAL ASYMP.

b) $\lim_{x \rightarrow -\infty} \frac{10x}{\sqrt{4x^2 + 1}} = \lim_{x \rightarrow -\infty} \frac{10x}{\sqrt{x^2} \sqrt{4 + \frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{10x}{|x| \sqrt{4 + \frac{1}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{10x}{(-x) \sqrt{4 + \frac{1}{x^2}}}$
 $= \lim_{x \rightarrow -\infty} \frac{10}{-\sqrt{4 + \frac{1}{x^2}}} = \frac{10}{-2} = -5$, so $y=-5$ is ALSO A HORIZONTAL ASYMP.