

A)

$$\begin{aligned} 1) \int e^{kx} dx &= \frac{1}{k} e^{kx} + C \\ 2) \int \cos kx dx &= \frac{1}{k} \sin kx + C \\ 3) \int \sin kx dx &= -\frac{1}{k} \cos kx + C \end{aligned}$$

FOR ANY CONSTANT  $k \neq 0$ .

$$\text{LET } \underline{u = kx}, \quad du = k dx$$

B) LOG RULE

$$\int \frac{g'(x)}{g(x)} dx = \ln |g(x)| + C$$

$$\text{LET } \underline{u = g(x)}, \quad du = g'(x) dx$$

C) EXPONENTIAL RULE

$$\int e^{g(x)} g'(x) dx = e^{g(x)} + C$$

$$\text{LET } \underline{u = g(x)}, \quad du = g'(x) dx$$

D) ARCTANGENT FORMULA

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

FOR ANY CONSTANT  $a > 0$ ,

$$\text{LET } \underline{u = \frac{x}{a}}, \quad x = au, \quad dx = a du$$

E) ARCSINE FORMULA

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

FOR ANY CONSTANT  $a > 0$ ,

$$\text{LET } \underline{u = \frac{x}{a}}, \quad x = au, \quad dx = a du$$

$$A) 1) \int e^{kx} dx \quad \text{Let } u = kx, \text{ so } du = k dx$$

$$= \frac{1}{k} \int e^{kx} \cdot k dx = \frac{1}{k} \int e^u du = \frac{1}{k} (e^u) + C = \boxed{\frac{1}{k} e^{kx} + C}$$

$$2) \int \sin kx dx \quad \text{Let } u = kx, \text{ so } du = k dx$$

$$= \frac{1}{k} \int \sin kx \cdot k dx = \frac{1}{k} \int \sin u du = \frac{1}{k} (-\cos u) + C = \boxed{-\frac{1}{k} \cos kx + C}$$

$$B) \int \frac{g'(x)}{g(x)} dx \quad \text{Let } u = g(x), \text{ so } du = g'(x) dx$$

$$= \int \frac{1}{g(x)} \cdot g'(x) dx = \int \frac{1}{u} du = \ln|u| + C = \boxed{\ln|g(x)| + C}$$

$$C) \int e^{g(x)} g'(x) dx \quad \text{Let } u = g(x), \text{ so } du = g'(x) dx$$

$$= \int e^u du = e^u + C = \boxed{e^{g(x)} + C}$$

$$D) \int \frac{1}{x^2 + a^2} dx \quad \text{Let } u = \frac{x}{a}, \text{ so } x = au \text{ and } dx = a du$$

$$= \int \frac{1}{a^2 u^2 + a^2} \cdot a du = \int \frac{1}{a^2(u^2 + 1)} \cdot a du = \frac{1}{a} \int \frac{1}{u^2 + 1} du = \frac{1}{a} \tan^{-1} u + C$$

$$= \boxed{\frac{1}{a} \tan^{-1} \frac{x}{a} + C}$$

$$E) \int \frac{1}{\sqrt{a^2 - x^2}} dx \quad \text{Let } u = \frac{x}{a}, \text{ so } x = au \text{ and } dx = a du$$

$$= \int \frac{1}{\sqrt{a^2 - a^2 u^2}} \cdot a du = \int \frac{1}{\sqrt{a^2(1 - u^2)}} \cdot a du = \int \frac{1}{\sqrt{a^2} \sqrt{1 - u^2}} \cdot a du$$

$$= \int \frac{1}{\sqrt{1 - u^2}} du = \sin^{-1} u + C = \boxed{\sin^{-1} \frac{x}{a} + C}$$

(where  $\sqrt{a^2} = |a| = a$   
for  $a > 0$ )

$$A) 2) \int \cos kx dx \quad \text{Let } u = kx, \text{ so } du = k dx$$

$$= \frac{1}{k} \int \cos kx \cdot k dx = \frac{1}{k} \int \cos u du = \frac{1}{k} (\sin u) + C = \boxed{\frac{1}{k} \sin kx + C}$$